Dark Pool Trading Strategies[*]

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October, 2011

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ABSTRACT

We model a financial market where traders have access both to a fully transparent limit order book (LOB) and to an opaque Dark Pool (DP). When a DP is introduced to a LOB market, orders migrate to the DP from the LOB, but overall trading volume increases. Moreover, inside quoted depth in the LOB decreases, but quoted spreads tend to narrow in deep books and widen in shallow ones. DP market share is higher when LOB depth is high, when LOB spread is narrow, when the tick size is large and when traders seek protection from price impact. When depth decreases on one side of the LOB, liquidity is drained from the DP. When Flash orders provide select traders with information about the state of the DP, more orders migrate from the LOB to the DP but overall market quality improves.

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According to the U.S. Securities and Exchange Commission (SEC), Dark Pools (DP) are Alternative Trading Systems (ATS) that do not provide their best-priced orders for inclusion in the consolidated quotation data. DP offer trading services to institutional investors that try to trade in size while minimizing adverse price impact. While undisplayed liquidity has always been a feature of U.S. equity markets, it is only recently that DP have been singled out for regulatory scrutiny. In 2009, the SEC proposed DP-related rule changes ranging from a ban of Flash orders to increased pre- and post-trade transparency for DP venues. Moreover, the recent *SEC 2010 Concept Release on Equity Market Structure* shows concerns on the effect of undisplayed liquidity on market quality as well as on fair access to sources of undisplayed liquidity.

Unfortunately, there is to date very limited academic research that sheds light on these issues. Existing models focus on the comparison between a dealer market and a crossing network (e.g. Degryse, Van Achter and Wuyts, 2009), thus overlooking the features that drive the strategic interaction of DP with limit order books (LOB). We extend this literature by building a theoretical model of a limit order market where traders can choose to submit orders either to the fully transparent LOB or to a DP. We derive the optimal trading strategies and characterize the resulting market equilibrium. Specifically, we show how stock liquidity, tick size and price pressure affect DP market share. We also demonstrate how the introduction of a DP affects overall trading volume and LOB measures of market quality. Finally, in an extension of our model, we show how Flash orders affect DP market share and market quality.

There are over thirty active DP in U.S. equity markets according to the SEC. A growing number of DP also operates in European equity markets. DP are characterized by limited or no pre-trade transparency, anonymity and derivative (almost exclusively mid-quote) pricing.

However, they differ in terms of whether or not they attract order flow through Indications Of Interest (IOI)[1] and whether or not they allow interaction with proprietary and black box order flow. DP report their executed trades in the consolidated trade data, but the trade reports are still not required to identify the ATS that executed the trade. As a result, it is difficult to accurately measure DP trading activity. Recent estimates suggest that DP represent over 12% of matched volume (*Rosenblatt Securities*, February 2011). As illustrated in Figure 1, there are four broad categories of DP, namely Public Crossing Networks, Internalization Pools, Exchange-Based Pools and Consortium-Based Pools.[2]

[**Insert Figure 1 here**]

As mentioned above, the SEC has raised several concerns associated with DP growth. A main concern is the possible migration of volume from transparent to dark markets, and hence the effect of DP trading on market quality. Another concern is the fair access to DP liquidity. While there are several aspects of the fair access issue, the problem that the SEC has focused on in their rule making is IOI messages. Indeed "actionable IOI" messages[3] work as public quotes with implicit pricing and, by creating a leakage of privileged information to select investors, they can unfairly discriminate against public investors. Hence on October 21, 2009 SEC Chairman Schapiro noted that "DP now represent a significant source of liquidity in U.S. stocks", creating a "two-tiered market", and announced a Proposal on Regulation of Non-Public Trading Interest (SEC Release No. 34-60997) for DP regulatory change. The

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[1]IOI are sales messages reflecting an indication of interest to either buy or sell securities. They can contain security names, prices and order size.

[2]Exchanges offer dark liquidity facilities that represent another 3.44% of matched volume (*Rosenblatt Securities*, February 2011).

[3]According to the SEC (2009), IOI messages are "actionable" if they explicitly or implicitly convey information on: the security's name, the side of the order, a price that is equal or better than the NBBO, and a size that is at least equal to one round lot. IOI are typically targeted to specific institutional customers and not broadcasted more widely.

aim of the proposal is to increase DP visibility by reducing the proportion of dark volume and by regulating IOI messages more severely.[4]

We use our model of a dynamic limit order market with a DP to shed light on these concerns. We find that the introduction of a DP induces order migration from the LOB to the DP but that overall trading volume increases. We also find that there is a positive liquidity externality in the DP so that DP orders beget more DP orders. We then study the factors that drive DP trading: DP market share is higher for stocks with higher inside order book depth and for stocks with narrower order book spreads. Intuitively, this can be explained as follows. Traders optimally trade off the execution uncertainty and the midquote price in the DP against the trading opportunities in the LOB. For stocks with higher depth at the inside or narrower spread, an order submitted to the LOB has to be more aggressive to gain priority over the existing orders in the order book. As a result, the alternative of a midquote execution in the DP becomes relatively more attractive. We also demonstrate that DP market share is higher when the tick size is larger. This follows since the wider inside spread makes market orders more expensive and hence DP orders a more attractive alternative. Finally, our model shows that traders use DP orders to reduce the price impact of their large orders. In particular we show that when large orders generate price pressure, traders either reduce their order size or resort to DP orders.

We also use our model to study the effect of DP trading activity on LOB market quality. We find that inside quoted depth and volume in the LOB always decrease when a DP is

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[4]More precisely, the SEC proposal addresses 3 issues: 1) actionable IOI: amendment of the definition of "bid" or "offer" in Rule 600(b)(8) of Regulation NMS to apply explicitly to actionable IOI, and exclusion of "size-discovery IOI", i.e. actionable who are reasonably believed to represent current contra-side trading interest of at least $200,000; 2) lower substantially the trading volume threshold (from the current 5% to 0.25%) in Rule 301(b) of Regulation ATS that triggers the obligation for ATS to display their best-priced orders in the consolidated quotation data; 3) require real-time disclosure of the identity of ATS on the reports of their executed trades.

introduced because orders migrate from the LOB to the DP. However, total volume in the LOB and DP combined actually increases. We also find that the introduction of a DP is associated with tighter quoted spreads when the book is deep but wider quoted spreads when it is shallow. The explanation is subtle and takes into account both the migration of orders to the DP and the switch between limit and market orders in the LOB. When the initial LOB depth is high, both market and limit orders switch to the DP, leaving spreads tight in the LOB. By contrast, when the initial LOB depth is low, competition from DP decreases limit orders execution probability and hence increases the use of market orders, thus widening the spread. Furthermore, we analyze the dynamic pattern in order flow: differently from Parlour (1998), only deep books exhibit a probability of continuation which is higher than that of a reversal, and the opposite holds for shallow books. Also, we find an externality originating from the interaction of a LOB with a DP, whereby the latter acts as a liquidity buffer.

Finally, we use our model to understand how introducing IOI messages such as Flash orders affects the equilibrium. We model Flash orders as a mechanism that provides select traders with information about the state of the DP before they submit orders. In this setting, we show that more orders migrate from the LOB to the DP, the reason being that everyone knows that informed institutions will use the DP. This means that the execution probability of DP orders increases, which reinforces the already existing liquidity externality effect. As a consequence, Flash orders have the overall effect of enhancing market quality. Indeed, compared to the market without asymmetric information, private information on the state of the DP reduces the execution risk of DP trading, thus making market orders less competitive than DP orders. The result is an improvement of both order book spread and depth, a reduction of LOB volumes but a further increase of total trading volume. As expected, we find that if more traders have access to IOI messages the beneficial effects of

IOI messages on market quality are even larger.

The paper is organized as follows. Section I reviews the related literature. Section II presents the general framework of the model, while in Section III the benchmark cases with both a Limit Order Book (LOB), a Dealer Market (DM) a well as the protocols with a DP are discussed and the equilibrium derived. Section IV reports results on market quality and on the dynamic pattern in order flow. In Section V, we extend the model to include asymmetric information on the state of the DP. Section VI discusses the model's empirical and policy implications and Section VII summarizes the results. All proofs are in the Appendix.

I   Literature on Dark Pools

The existing theory on undisplayed liquidity focuses on the interaction between crossing networks (CN) and dealer markets. Hendershott and Mendelson (2000) model the interaction between a CN and a DM and show costs and benefits of order flow fragmentation. Donges and Heinemann (2004) model intermarket competition as a coordination game among traders and investigate when a DM and a CN can coexist; Foster, Gervais and Ramaswamy (2007) show that a volume-conditional order-crossing mechanism next to a DM market Pareto improves the welfare of additional traders. The model we propose differs from these as it considers the interaction between a LOB and a DP rather than a DM and a CN; furthermore, it focuses on the dynamic, rather than static, order submission strategies of traders. More recently, Ye (2009) uses Kyle's model to find the insider's optimal strategic use of a DP and to show that DP harm price discovery especially for stocks with high volatility; however Ye assumes that only the insider can strategically opt to trade in the DP and he models uninformed traders as noise traders. An opposite result on price discovery is obtained by Zhu (2011)

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who models the interaction among insiders, constrained and unconstrained noise traders by using a Glosten and Milgrom (1985) type framework. Kratz and Schoeneborn (2009) prove existence and uniqueness of optimal trading strategies for a trader who can split orders between an exchange and a DP, but assume that the price impact and the DP's liquidity are exogenously given.

The paper which is closest to ours is that by Degryse, Van Achter, and Wuyts (DVW, 2009), who investigate the interaction of a CN and a DM and show that the composition and dynamics of the order flow on both systems depend on the level of transparency. Our paper differs from DVW (2009) in that it considers the interaction between a LOB -rather than a DM- and a DP: this means that in our model traders can use both market orders and limit orders, and it is precisely the effect of competition from limit orders that drives the results we obtain compared to those of DVW.

Empirical work on crossing networks is relatively limited. Gresse (2006) finds that POSIT's crossing network has a market share of one to two percent of share volume and, by investigating the relation between the CN trading and the liquidity of the SEAQ quote-driven segment of the LSE, finds no negative effect of the CN on the dealership market. Gresse (2006) also finds that there is no significant increase in adverse selection or inventory risk, but rather a spread decrease due to increased competition and risk sharing. Conrad, Johnson, and Wahal (2003) find that realized execution costs are generally lower on Alternative Trading Systems and that institutional orders sent to traditional brokers have higher execution costs than those executed in the CN. Naes and Odegaard (2006) provide evidence that orders from large institutional investors have lower realized execution costs for the component of the orders sent to the CN, but higher costs of delay if one considers the entire orders and includes the component sent to standard exchanges. Fong, Madhavan and

Swan (2004) find no evidence that competition from the upstairs market and the CN has an adverse effect on the limit order book of the Australian Stock Exchange.

To our knowledge, there is still limited empirical analysis on DP in the academic literature. Ready (2009) studies monthly volume by stock in three DP: Liquidnet, POSIT, and Pipeline during the period June 2005 to September 2007. The data suggests that these three DP execute roughly 2.5 percent of consolidated volume (third quarter 2007) in stocks where they were active during a month, but only 1 percent of market consolidated volume. He finds that DP execute most of their volume in liquid stocks (low spreads, high share volume), but they execute the smallest fraction of share of volume in those same stocks. Buti, Rindi and Werner (2011) examine a unique dataset on dark pool activity for a large cross section of US securities and find that liquid stocks are characterized by more dark pool activity. They also find that dark pool volume increases for stocks with narrow quoted spreads and high inside bid depth, suggesting that a higher degree of competition in the limit order book enhances DP activity. Finally, Degryse, de Jong and van Kervel (2011) consider a sample of 52 Dutch stocks and analyze both trades internalized on crossing networks and trades sent to dark pools. They find that when these two sources of dark liquidity are combined, the overall effect on global liquidity is detrimental.

Other strands of the academic literature are relevant for understanding the role of DP in today's markets. DP are characterized by limited or no pre-trade transparency, and issues of anonymity and transparency are therefore important.[5] DP also coexist with more transparent venues, which implies a link with the literature on multimarket trading.[6] Finally,

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[5]See for example the theoretical works by Admati and Pfleiderer (1991), Baruch (2005), Fishman and Longstaff (1992), Forster and George (1992), Madhavan (1995), Pagano and Röell (1996), Röell (1991), and Theissen (2001). Several empirical papers have recently explored the significance of anonymity and transparency in experimental settings and real data: Bloomfield and O'Hara (1999, 2000), Boehmer, Saar, and Yu (2005), Flood, Huisman, Koedijk and Mahieu (1999) and Foucault, Moinas and Theissen (2007).

[6]See among the others: Barclay, Hendershott, and McCormick (2003), Baruch, Karolyi and Lemmon

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DP are currently competing with other dark options offered by exchanges to market participants, which builds a connection with the recent literature on hidden orders. In this regard, Buti and Rindi (2011) analyze a new type of totally undisclosed orders, Hidden Mid-Point Pegged, that are posted at the spread midquote and hence are the most natural competitors with orders submitted to the DP. On the empirical side, Bessembinder, Panayides and Venkataraman (2009) study the costs and benefits of iceberg orders at Euronext and find that these orders bear smaller implementation shortfall costs, thus suggesting that, similarly to DP, they provide a protection from price impact.[7]

II The Model

In this Section we present a model for three different market structures. We start with a limit order book with both retail and institutional traders and use it as a benchmark model. We then add a Dark Pool which allows us to consider a market structure where traders can choose between the two platforms. Finally, we study the market structure formed by both a dealership market and a DP, that has extensively been modelled by previous literature (e.g. Degryse, Van Achter, and Wuyts, 2009). We find remarkable differences when we compare this market structure with the LOB plus DP mechanism.

(2007), Bennett and Wei (2006), Bessembinder and Kaufman (1997), Boehmer and Boehmer (2003), Easley, Kiefer, and O'Hara (1996), Foucault and Menkveld (2008), Goldstein, Shkilko, Van Ness and Van Ness (2008), Karolyi (2006), Lee (1993), Nguyen, Van Ness, and Van Ness (2007), Pagano (1989), Reiss and Werner (2004) and Subrahmanyam (1997).

[7]Other theoretical works on hidden orders are Esser and Mönch (2007) and Moinas (2006). The empirical literature is instead more extensive, other contributions are De Winne and D'Hondt (2007), Frey and Sandas (2008), Hasbrouck and Saar (2004), Pardo and Pascual (2006) and Tuttle (2006).

A Market Structure

We consider a discrete time protocol that, as in Parlour (1998), features a limit order book for a security which pays v at each period and is assumed constant through the trading game. Trading occurs during a day that is divided into T periods: $t = 1, ..., T$. In each period t a new risk neutral trader arrives who can be with equal probability either a large institutional trader or a small retail trader. Large traders can trade $j = [0, 2]$ shares, whereas small traders can only trade up to 1 share at a time. Upon arrival at the market the trader selects both a trading venue and an order type, and his optimal trading strategy cannot be modified thereafter: small traders can only trade in the LOB, while large traders can choose to trade either in the LOB or in the DP.[8] Traders' personal valuation of the asset is represented by a multiplicative parameter, β_t, drawn from a uniform distribution with support $[0, 2]$: traders with a high value of β are impatient to buy the asset, while traders with a low β value the asset very little and therefore are impatient to sell it; traders with a β next to 1 are patient as their evaluation of the asset is close to the common value.

The LOB is characterized by a set of four prices and associated quantities, denoted as $\{p_i^B \& q_i^B, p_i^A \& q_i^A\}$, where A (B) indicates the ask (bid) side of the market and $i = \{1, 2\}$ the level on the price grid. Hence, prices are defined relative to the common value of the asset, v:

$$
\begin{aligned}
p_i^A &= v + i\,\tau \\
p_i^B &= v - i\,\tau
\end{aligned}
$$

where τ is the minimum price increment that traders are allowed to quote over the existing

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[8]In this model DP are designed to trade large blocks. For this reason, we do not allow either small traders to post their orders to the DP, or large traders to split their orders between the DP and the LOB.

price, and hence it is the minimum spread that can prevail on the LOB. The associated quantities denote the number of shares that are available at that price. Following Parlour (1998) and Seppi (1997), we assume that a trading crowd absorbs whatever amount of the risky asset is demanded or offered at p_2^A and p_2^B. Hence at the second level of the book depth is unlimited and traders can only demand liquidity, whereas the number of shares available at p_1^A (p_1^B) forms the state of the book that characterizes time t and is defined as $b_t = [q_1^A q_1^B]$.

The DP operates next to the LOB; it allows market participants to enter unpriced orders to buy or sell the asset, and it is organized as a crossing network where time priority is enforced. In this trading venue orders are crossed at the end of time T at the spread midquote prevailing on the LOB in that period, p_{Mid}. If a trader submits an order to the DP, this order will be executed provided that there is sufficient depth to match it. The novelty of the DP compared to the standard crossing networks, however, is that traders are unable to observe the orders previously submitted by the other market participants. It follows that they can only infer DP depth by monitoring the LOB.

As will be discussed more in detail below, we only consider the last three periods of the trading game and we assume that at $T - 2$ agents assign equal probabilities to the following three states of the DP's depth:

$$
DP_{T-2} = \begin{cases} +6 & with \ prob = \frac{1}{3} \\ \ 0 & with \ prob = \frac{1}{3} \\ -6 & with \ prob = \frac{1}{3} \end{cases} \tag{1}
$$

This means that at time $T - 2$ traders believe that either the DP is empty, or that it is full on one or the other side of the market.[9] We also assume that traders monitor the book

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[9]As at $T - 2$ there are only three periods left in the trading game, if for example six shares to sell are

and that when they do not observe any market or limit orders, they Bayesian update their expectations on the state of the DP. Hence, traders at T face uncertainty from two sources as they have to make inference on the state of the DP at both $T-2$ and $T-1$.

It is straightforward to extend the model discussed so far to include a DM that competes with a DP. Technically, this can be accomplished by moving the trading crowd to the first level of the LOB. Note that the resulting market structure is in this case identical to the one discussed in DVW (2009).

B   Order Submission Strategies

Upon arrival at the market each trader decides his optimal trading venue as well as the optimal order type. To this end he compares the expected profits from the different order types he can choose. The feasibility and profitability of these orders depend on the traders' type (β_t) as well as on both the state of the LOB (b_t) and the state of the DP $(\widetilde{DP_t})$ at the time of the order submission.

Consider for example the sell side. If the incoming trader opts for a market sell order of size j, $\varphi(j, \overline{p}_i^B)$,[10] he will pay the spread but his order will execute and his payoff will be:

$$\pi_t[\varphi(j, \overline{p}_i^B)] = (\overline{p}_i^B - \beta_t v)\, j$$

If instead j exceeds the depth at the highest bid-price in the book, an impatient trader may submit a marketable order that will walk down the LOB hitting different prices, $\varphi(2, \overline{p}^B)$,

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already standing on the ask side of the DP, $DP_{T-2} = [-6]$, the execution probability of any other share posted to the ask side is zero, the reason being that at the most two shares can be executed at each trading round.

    [10]Notice that we indicate with an upper bar the prices at which market orders are executed. Clearly these execution prices will be determined by the state of the LOB.

to complete its execution:[11]

$$\pi_t[\varphi(2,\overline{p}^B)] = (\overline{p}_1^B + \overline{p}_2^B) - 2\beta_t v$$

A more patient trader can instead submit a limit sell order of size j to p_1^A, $\varphi(j,p_1^A)$, seeking to obtain a higher sale price at the cost of an uncertain execution. In this case, his profits will depend on the probability of the order being executed in the following trading rounds, from time $t+1$ to T:

$$\pi_t^e[\varphi(j,p_1^A)] = (p_1^A - \beta_t v)E\{\sum_{w_{t+1}=1,j} w_{t+1}\Pr_{w_{t+1}}(p_1^A|\Omega_{t+1})+$$

$$[\sum_{l=t+2}^{T}\sum_{W=0}^{1}\sum_{w_l=1}^{j-W} w_l\Pr_{w_l}(p_1^A|\Omega_l)\Pr(\sum_{m=t+1}^{l-1} w_m = W|\Omega_{l-1})]\}$$

where $\Omega_t = \{b_t, v, \widetilde{DP}_t\}$, $\Pr_{w_l}(p_1^A|\Omega_l)$ is the probability that w_l shares will be executed at $t = l$, and W is the number of shares executed up to $t = l - 1$.

If the incoming trader is large, he may alternatively submit a j-order to sell to the DP, $\varphi(-j,p_{Mid})$, that can be executed at the end of the trading game at the prevailing spread midpoint. Hence, for this order type not only the execution probability is uncertain, but also the execution price. This strategy has the following expected payoff:

$$\pi_t^e[\varphi(-j,p_{Mid})] = E[(p_{Mid} - \beta_t v)\Pr_{-j}(p_{Mid}|\Omega_T)]\, j$$

where $\Pr_{-j}(p_{Mid}|\Omega_T)$ is the probability that j shares to sell will be executed in the DP. Finally the trader can decide not to trade at all. In this case his payoff will be equal to zero, $\pi_t[\varphi(0)] = 0$. The strategies on the buy side of the market are symmetric and are left out for brevity.

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[11]We omit the subscript i for the level of the book since the order will be executed at different prices. Notice also that in this case j will clearly be equal to 2.

III Market for Liquidity

The model is solved under three specifications that correspond to three different market structures. First, we present a benchmark model that describes the working of a pure LOB; then we focus on the protocol with a LOB competing with a DP (LOB&DP) and we compare the results obtained with the case where a DM, rather than a LOB, competes with the DP (DM&DP). This analysis allows us to discuss the driving factors of DP trading, and the effects that the price impact generated by large trades can have on traders' choice between a LOB and a DP.

A Benchmark Model

We focus on a three-period trading game that ends at T. At each period nature selects a new small or large trader with equal probability. Figure 2 shows an example of the extensive form of the trading game: the market opens at $T-2$ with two units on the best bid and offer, $b_{T-2} = [22]$. Given this state of the book, the equilibrium strategies at $T-2$ for both large and small traders include a limit order at either the best ask or the best bid, or a market order that hits the limit order standing at the first level of the book. Suppose that at $T-2$ nature selects a small trader who is rather patient and decides to submit a limit order at p_1^A, $\varphi(1, p_1^A)$; then at $T-1$ the book will open with 3 units on p_1^A. If at $T-1$ nature selects another small trader who decides not to trade, then the book will open unchanged at T and the new incoming trader will choose among market buy, market sell and no trade. Notice that traders do not submit limit orders at time T as the market closes and hence their execution probability is zero.

[**Insert Figure 2 here**]

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For each trading round, the arriving risk-neutral large trader will choose the optimal order submission strategy which maximizes his expected profits conditional on the state of the LOB, b_t, and his type, β_t. A large trader (LT) thus chooses:

$$\max_{\varphi} \pi_t^e[\varphi(j, \overline{p}_i^B), \varphi(2, \overline{p}^B), \varphi(j, p_1^A), \varphi(j, \overline{p}_i^A), \varphi(2, \overline{p}^A), \varphi(j, p_1^B), \varphi(0) \mid \beta_t, b_t] \qquad (2)$$

and a small trader (ST) chooses:

$$\max_{\varphi} \pi_t^e[\varphi(1, \overline{p}_i^B), \varphi(1, p_1^A), \varphi(1, \overline{p}_i^A), \varphi(1, p_1^B), \varphi(0) \mid \beta_t, b_t] \qquad (3)$$

We find the solution of this game by backward induction and by assuming that the tick size, τ, is equal to 0.1. We start from the end-nodes at time T and compare trading profits for both large and small traders. This allows us to determine the probability of the equilibrium trading strategies at T that can be market orders, as well as no trading. We can hence calculate the execution probabilities of limit orders placed at $T-1$. This in turn allows us to compute the equilibrium order submission strategies in that period. Given the probability of market orders at $T-1$, we can finally compute the equilibrium order submission strategies at $T-2$.

The framework described so far can also be simplified to analyze a pure dealership market. This can be accomplished by moving the trading crowd to the first level of the book, thus limiting traders to only submit market orders or not to trade as in DVW (2009).

B   Intermarket Competition: LOB&DP vs. DM&DP

Once a DP is added to the LOB, large traders have the additional option to submit an order to buy or to sell to the DP. Provided that there will be enough depth to match it, the order

will be executed at the end of time T. As shown in Figure 3, all else equal, now this new order type is added to the strategies of large traders in the extensive form of the trading game.

[**Insert Figure 3 here**]

As in the previous case, at each trading round the risk-neutral large trader chooses the optimal order submission strategy which maximizes his expected profits. However, in this new framework he conditions not only on the state of the LOB, b_t, and his type, β_t, but also on the state of the DP, $\widetilde{DP}_t$. A large trader thus chooses the order that leads to the largest profits:

$$\max_{\varphi} \ \pi_t^e[\varphi(j,\overline{p}_i^B),\varphi(2,\overline{p}^B),\varphi(j,p_1^A),\varphi(j,\overline{p}_i^A),\varphi(2,\overline{p}^A),\varphi(j,p_1^B),\varphi(\pm j,p_{Mid}),\varphi(0) \ |\beta_t,b_t,\widetilde{DP}_t] \quad (4)$$

Small traders still solve problem (3), however they will now condition their strategies not only on their type and on the state of the LOB, but also on the state of the DP.

If instead a DP is added to a DM, the optimization problem for large traders simplifies to:

$$\max_{\varphi} \ \pi_t^e[\varphi(j,\overline{p}_1^B),\varphi(j,\overline{p}_1^A),\varphi(\pm j,p_{Mid}),\varphi(0) \ |\beta_t,\widetilde{DP}_t] \quad (5)$$

as in the DM&DP protocol traders cannot submit limit orders (Figure 4).

[**Insert Figure 4 here**]

A relevant issue in market design is to establish whether by adding a new trading opportunity to a limit order book more volume is created, or whether volume is simply diverted to the new trading venue. The results from this model show that when a DP is added to

a LOB, volumes shift to the DP and there is no trade creation. Conversely, when a DP is added to a DM, it indeed induces some traders to enter the market. The latter result replicates the case studied by DVW (2009). The intuition is rather simple: in the dealership market some patient traders, who do not have the possibility to compete for the provision of liquidity by using limit orders, refrain from trading to avoid paying the spread. However, when they are offered the opportunity to submit orders to the DP, they take this option as they can eventually profit from obtaining execution at the midquote. By contrast, in the LOB there is no such effect as patient traders are already allowed to submit limit orders. The following Proposition summarizes the results obtained by comparing the two protocols.

Proposition 1 .

- *When a DP is added to a LOB, it induces order migration to the dark market. When instead a DP is added to a DM, it produces trade creation.*

- *Order migration is more intense when the book is tight and competition for the provision of liquidity is strong.*

- *DP generate a liquidity-externality effect: as existing dark liquidity begets future liquidity, it increases the execution probability of dark orders*

Table I reports results on equilibrium trading strategies of large traders for $b_{(T-2,T-1,T)} =$ [22]. Notice that when at $t = T$ the book opens with two shares at the inside spread, the LOB&DP framework converges to the DM&DP one. Indeed, given the maximum trade size of two shares, the LOB is full and works like a DM where dealers offer unlimited liquidity at the BBO (as in DVW, 2009). However, in the earlier periods traders can compete for the provision of liquidity by submitting limit orders to the LOB, and the role of these orders is

crucial to understand the differences between the LOB&DP and the DM&DP frameworks. Clearly, the longer the time to the end of the trading game, the more relevant are limit orders, as their execution probability increases. Hence, the comparison between the equilibrium trading strategies at $T - 2$ and those at T best captures the differences between the two frameworks.

Our results for the sell side at $T - 2$ show that traders in a LOB&DP compared to the LOB submit fewer limit orders (0.0109 compared to 0.0314) and fewer market orders (0.4612 compared to 0.4686), as they opt for DP orders with probability 0.0279 (Table I). This means that there is no trade creation but only order migration to the DP. The same comparative static exercise performed at time T, when traders cannot submit limit orders, results in trade creation exactly as in DVW (2009). Actually at T, when a DP option is offered to market participants, those traders who were not willing to enter the market, move to the DP with probability .0375. However, as discussed above, at T the LOB converges to the DM and hence trade creation takes place because in a DM traders cannot submit limit orders. Same results are obtained for the buy side.

[**Insert Table I here**]

The overall effect of intermarket competition also depends on the state of the LOB. Table II reports results obtained by assuming that at $T - 2$ the LOB opens empty. Clearly, when the LOB is empty, limit orders are more attractive as traders can post these orders at the top of the queue and they therefore have a higher execution probability. Hence, when the LOB is empty at $T - 2$, competition from limit order is so intense that it crowds out the DP. In this case, when traders are allowed to choose between a LOB and a DP, they opt for the former. It follows that trade migration to DP is greater when depth builds up in the

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LOB.[12]

<div align="center">[**Insert Table II here**]</div>

Finally Table I shows that traders' perception of DP liquidity influences the execution probability of DP orders and hence their use. When at $T-2$ traders do not observe any change in the LOB, they assume that either no trade occurred or that an order was submitted to the DP. As they perceive that liquidity is building in the DP, they update their estimate of the DP depth and assign a higher probability of execution to their DP orders, the result being that they opt for DP more frequently. As an example, this effect can be observed by comparing the results for $T-1$ presented in Table I for the case "vis_{T-2}", where traders observe a change in the LOB at $T-2$, and "inv_{T-2}", where instead traders observe no change. In the latter case, they submit orders to the DP more intensively (.050) than when they have no uncertainty (.0379). Analogous results are shown for the DM&DP market where the probability of DP orders increases from .0379 to .0444 when traders do not observe any change in the LOB at $T-2$. We can therefore conclude that the positive liquidity-externality effect produced by a DP intensifies when traders perceive that DP volume is growing. This is consistent with the empirical results by Buti, Rindi and Werner (2011) that show the existence of a positive auto-correlation between contemporaneous and lagged DP activity.

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[12] As mentioned before, we require a minimum trade size of 2 shares to access the DP. Had we allowed a DP trade size of 1 share, depending on the states of both the LOB and the DP, we would expect to observe that on the one side small traders should enter the DP, while on the other side large traders could start splitting their orders between the LOB and the DP. While we conjecture that our results should not change qualitatively, we cannot predict whether the DP will be used more or less extensively compared to the case where the DP minimum trade size is 2.

C   Dark Pool Drivers

We have shown that the state of the LOB affects traders' choice between transparent LOB and opaque DP trading. We now extend this analysis by discussing more in depth the main factors related to the state of the LOB, i.e. depth, spread and tick size, that affect traders' choice to submit orders to the dark market. The following Proposition summarizes the results.

Proposition 2 *The probability that traders submit orders to the DP:*

- *increases with market depth and the tick size, and*

- *decreases when the inside spread widens.*

  Opposite results relative to depth and inside spread are obtained for a DM&DP.

To investigate the effects of depth, spread and tick size on traders' choice, we compare, as we did before, the equilibrium strategies at $T - 2$ and at T. Once again, the longer the time to the end of the game, the higher is the probability of limit order execution and the stronger is the effect of limit order competition. Table III (Panel A) shows that at $T - 2$ an increase in depth on the top of the book from [11] to [22] reduces competition from limit orders and increases the probability that traders opt for the DP: $\varphi(2, p_1^A)$ and $\varphi(2, p_1^B)$ decrease (from .0832 to .0109) and, even though market orders increase from .4168 to .4612, traders now use the DP. If instead the same comparative static exercise is performed at time T, when there is no competition from limit orders and the LOB resembles a dealership market, we obtain the same result as in DVW (2009). Indeed, Table IV shows that when depth on the ask side increases from [11] to [21], market orders to buy increase from .4250 to .4625 and crowd out DP buy orders, which decrease from .0750 to .0375. The same results

are obtained when depth increases only on the bid side or on both sides of the market (from [11] to [12] or to [22]). We conclude that when market participants can compete for the provision of liquidity by using limit orders and can also opt for DP orders, the deeper is the limit order book, the longer is the queue for their limit orders (due to time priority) and the greater is the probability that they opt for DP orders. By contrast, competition from limit orders is absent in dealership markets and greater depth fosters traders' aggressiveness thus increasing market orders to the detriment of DP orders.

[**Insert Tables III and IV here**]

Our results from comparative statics on the inside spread confirm those from market depth. For the LOB&DP framework, the more liquid is the market the more competitive is the limit order book and the higher is the probability that the traders opt for the DP. This is consistent with Buti, Rindi and Werner (2011) who find that stocks with narrower quoted spreads have greater DP volumes, suggesting that DP are more active when the degree of competition in the LOB is high. To isolate the effect of variations in the spread, it is necessary to control for market depth. This can be accomplished by comparing two states of the book that have enough liquidity at the BBO to absorb large orders. Table III and IV show results for both time $T - 2$ and T. Starting again from period $T - 2$, when the inside spread increases, i.e. the state of the book changes from [22] to [00], the increased competition for liquidity provision crowds out DP orders, even though the probability of market orders decreases. We thus find that the wider the inside spread, the more convenient are limit orders submitted at the top of the book, and the greater is the probability that traders choose limit instead of DP orders. Opposite conclusions can be drawn from the same simulation performed at time T for the DM&DP framework. Table IV shows that when the spread increases (from [22] to e.g. [00]) competition from market orders decreases and,

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because at T there is no competition from limit orders, the probability that traders opt for DP increases from .0375 to .1125.

Proposition 2 also illustrates the effects of a change in the tick size on the probability of DP orders. When the tick size increases traders become more willing to supply liquidity. An example for the book [11] is shown in Table III where, following an increase in the tick size, market orders decrease, while limit and DP orders increase. The intuition for this result is that an increase in the tick size produces two effects: it widens the inside spread, and hence makes market orders more expensive, and it increases the minimum price change, thus making it more convenient for traders to supply liquidity. The end result is that more patient traders will opt for limit orders whereas less patient traders will chose to trade in the DP.

D   Price Impact, Price Pressure and Dark Pool Trading

One of the main reasons for why institutional traders submit orders to Dark Pools is to reduce price impact. A price impact can arise both when impatient traders submit a large order to the top of a LOB that is not deep enough to absorb the order, and when a patient trader submits a limit order that produces a price pressure, thus temporarily moving the asset value against the trader's order. Price impact resulting from trades has been extensively investigated: for example, Engle and Patton (2004) analyze the price impact of 100 NYSE stocks stratified by trade frequency and find strong evidence of short-run price impact for trades initiated by both buyers and sellers.[13] Price pressure[14] arising from passive order placement through limit orders has been recently explored by Hendershott and Menkveld (2010), who estimate the price impact arising from liquidity supply. They find a large daily transitory volatility in returns for stocks listed at the NYSE due to price pressure.

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[13] See also Hasbrouck (1991) and Dufour and Engle (2000).

[14] See Gabaix et al. (2006), Brunnermeier and Pedersen (2009) and Parlour and Seppi (2008).

In our model what drives large institutional traders to operate in a DP is their wish to buy or sell large blocks with the lowest price impact. Consider first impatient traders who are concerned about the price impact that can be generated by a market order. These traders face the standard trade-off between price risk (i.e. bearing a price impact) and execution risk. If they choose a market order, they will obtain immediate execution but will pay a greater price impact, which is increasing in the lack of depth available on the opposite side of the market. If instead they opt for the DP, their order will be executed with a lower price impact at the spread midquote. However, the order execution will be uncertain and will depend on the state of the DP.

Table IV shows that the impatient trader is more likely to opt for submitting a DP order when the other side of the market is shallow and the price impact therefore is large. For example, when the book opens with only 1 share on top of the ask side, $b_{T=}[12]$, instead of 2 shares, $b_{T=}[22]$, large traders use DP buy orders more intensively (with probability .0750 instead of .0375) as in the former case their order will move the price up to obtain execution.

To capture the price impact of limit orders submitted by patient traders in as simple a way as possible, we extend our model to embed the temporary price pressure that can be generated by these orders. More precisely, we assume that large limit orders produce a short term price pressure that lasts for one period, as shown in Figure 5.

[**Insert Figure 5 here**]

Suppose that a large seller arrives at the market at $T - 2$ and submits a large limit order at p_1^A. Following this submission, the asset value jumps down by 1 tick and the next period the market opens with $v_{T-1} = v - \tau$. Clearly at time T the temporary price effect vanishes and the asset value jumps back to v.[15] We consider two different specifications with

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[15]There are many elaborate ways to model price pressure, the way we do it is akin to temporary price

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price pressure: a benchmark model with no DP (LOB&PP) and a model that allows for DP trading (LOB&DP&PP). The results are summarized in the following Proposition.

Proposition 3 *When large orders generate price pressure, traders either reduce the size of their order, or, if available, switch to DP orders.*

Notice that when price pressure is introduced, the execution probability of a large limit order decreases for two reasons. First, the initial order is now on the second level of the book and hence further away from the asset value. Second, the initial order can easily be front-run in the following period by an incoming trader posting a limit order at the now empty first level of the book. The result is that traders switch to those order types that protect from price impact either because they are small in size, or because they are undisclosed. In Table V one can indeed notice that moving from the standard LOB protocol to the specification with price pressure, traders reduce their price impact by switching from large to small limit orders. When instead we introduce price pressure in the model with a DP, traders actually minimize their price impact by submitting DP buy and sell orders with larger probability (from .0279 to .0316).

[**Insert Table V here**]

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impact. Other possibilities would include, for example, assuming a permanent price impact, or one that lasts $t > 1$ periods. However, what is relevant here is not the duration of the price change but rather the existence of a price variation.

IV   Market Quality and Systematic Pattern in Order Flow

So far we have shown how traders react when a DP is added to a LOB and concluded that DP produce order migration. The next relevant issue is to investigate how the introduction of a DP affects the quality of the LOB, and how it impacts the dynamic pattern in order flow.

A   Market Quality

To evaluate the effect of DP trading on market quality, we consider inside spread (S_t), market depth (D_t) and volume (V_t). We compute expected spread and depth in period $t+1$ by weighing the values that characterize a particular state of the book with the corresponding order submission probabilities in the previous period:

$$y_{t+1}^e = \sum_{a=ST,LT} \Pr(a)\ E\left[\int_0^2 y_{t+1}(\varphi_a^n) \times f\left(\beta_t\right) d\beta_t\right] \tag{6}$$

where $y_{t+1} = \{S_{t+1}, D_{t+1}\}$ and φ_a^n with $n \in N_t$ are the equilibrium strategies of agent a at period t.

We calculate the expected LOB volume in each period t in a similar way and weigh by size the market orders submitted to the LOB:

$$V_t^e = \sum_{a=ST,LT} \Pr(a)\ E\left[\int_0^2 q_t(\varphi_a^n) \times f\left(\beta_t\right) d\beta_t\right] \tag{7}$$

where $q_t(\varphi_a^n)$ is the traded quantity which is a function of the agent's type a. Proposition 4

summarizes the results.

Proposition 4 *When a DP is added to a LOB, market quality changes as follows:*

- *market depth at the best bid-offer decreases;*

- *inside spread decreases when the LOB opens deep, the opposite holding when it opens empty;*

- *LOB volume decreases, whereas total volume increases.*

We have previously proved that the DP can attract orders from the LOB. Specifically, Table I shows that when the book opens deep ($b_{T-2} = [22]$) by moving from the LOB to the LOB&DP protocol, the probability of both limit and market orders decrease, as traders switch to the DP. Clearly the effect of the order migration on liquidity and volume depends on the proportion of limit vs market orders that leave the book. A reduction of the probability that traders post limit orders to the LOB decreases the provision of liquidity and hence leads to a reduction in the market depth and a widening of the inside spread. By contrast, a reduction of the demand for liquidity, i.e., the probability that traders submit market orders, certainly decreases volume but can have positive effects on both depth and inside spread as market orders subtract liquidity from the book.

Table VI shows that when the book opens with two shares on both sides, $b_{T-2} = [22]$, the introduction of the DP decreases average depth and volume but it improves the inside spread. This means that the positive effect on spread of the reduction of market orders more than outweighs the negative effect of the reduction of limit orders. The two opposite effects on liquidity arise because when the opening book is already very deep at the inside spread,

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orders that move to the DP leave the best bid-offer very tight.[16] When instead the book opens empty at $T-2$ or with only one share at the best bid-offer, all the three measures of liquidity worsen on average. In this case, the introduction of a DP makes limit orders less attractive so that traders opt for market orders, and as a result the inside spread increases (see for example Table II).

[**Insert Table VI here**]

Overall Proposition 4 shows that by moving from the LOB to the LOB&DP, depth and volume decrease, whereas the effect on the inside spread depends on the depth initially available at the top of the book. When the book is empty or has only 1 share available, then the migration also worsens the inside spread and the whole market quality deteriorates. When instead the book opens with 2 shares at the inside, then the effect on the average inside spread is positive and the overall effect on liquidity is mixed. Finally, Table VI shows that the overall effect of the introduction of a DP on total volume is positive. The sum of the LOB and DP volume is in fact systematically greater than the amount of volume traded in the benchmark LOB.

B   Systematic Pattern in Order Flow

Traders' strategic interaction with the two sides of the LOB and with the DP allows us to draw conclusions on the systematic pattern of the order flow, which are summarized in the following Proposition.

Proposition 5 *The following systematic pattern typifies order flows in the LOB and in the LOB&DP framework:*

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[16]Technically, when the book is deep and tight and market orders move to the DP, the probability that the spread remains small increases; the opposite happens when the initial spread is wide.

- *when the book is deep, the probability of a continuation is greater than that of a reversal, whereas when the book is shallow the opposite holds;*

- *the DP has a positive externality effect on the limit order book: if depth decreases on one side, competition for limit orders increases and liquidity gets drained from the DP to the LOB. Hence, volumes show a smaller decline in the LOB&DP protocol.*

Parlour (1998) shows that the interaction of traders with the two sides of the book entails a probability of a continuation greater than that of a reversal, and this is consistent with Biais, Hillion and Spatt (1999). We find that this effect only holds when the book is deep, whereas it is not supported by the model when the top of the book is shallow and traders have to walk up (or down) the book in search of execution. The difference in the results originates from the fact that in Parlour's LOB the trading crowd is positioned at the top of the book, whereas in our model there is a two-level price grid and the trading crowd is not posted at the first level, but rather at the highest (second) level. This means that in our model traders have to walk up (or down) the book when there is not enough liquidity at the top. An example will help understanding why the need to walk up the book entails a probability of a continuation smaller than that of a reversal.

Consider Table VII where the equilibrium strategies at $T-2$ are reported for two different states of both the LOB and the LOB&DP, $b_{T-2} = [20]$ and $b_{T-2} = [10]$. Comparison between these two books allows us to compute the equilibrium trading strategies of a buyer arriving at the market at $T-2$ and facing a book either with 2 shares at the best ask, or with only 1 share. The latter state of the book can occur if, for example, at time $T-3$ the book opens with 2 units [20] and a market buy order arrives leaving the book with only 1 share on the ask side. Consider first a small buyer who has to decide whether to submit a limit or a market order. The observed reduction of the depth on the opposite side of the book informs

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him that future sellers will rather post a limit order to sell than a market order to sell:[17] Table VII shows for example that the probability of observing limit sell orders increases in percentage by 1.950 and by 0.8120 respectively for the two cases with and without a DP, and that the probability of observing market sell orders decreases by 0.0857 and 0.1206. This shift from market to limit sell orders implies that the probability of execution of any eventual limit order to buy decreases, thus inducing the small trader to submit more market than limit buy orders. As a result, the continuation probability of a small buy order becomes greater than that of a reversal. Indeed, Table VII shows that, after observing a reduction of the depth at the best ask due to a market buy order, the probability that a small trader at $T-2$ submits another market buy order increases by .0128 and .0151 respectively in the market with and without a DP.

[**Insert Table VII here**]

Notice that for the small trader in both cases the top of the ask side of the book is deep enough to have a buy order executed without walking up the book. If instead we consider the choice of a large buyer arriving at the market at $T-2$, we observe that in a book [10], despite the lower execution probability of a limit buy order, he will submit fewer rather than more market buy orders, thus increasing the probability of price reversal. The reason is that when the book changes from [20] to [10], the large trader will have to walk up the book to have his order executed, thus paying a higher price. As the reversal effect for large traders is stronger than the continuation effect for small traders, the average probability of a continuation is smaller than that of a reversal for both the LOB and the LOB&DP markets: $-.0739$ and $-.0705$. Clearly, if after the arrival of a market buy order the final state of

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[17]Here we refer to the average probability of limit orders and market orders submitted by both large and small traders.

the book at $T-2$ were still deep enough even for a large trader (e.g. moving from [30] to [20]), then the 'Parlour effect' would still hold and the probability of a continuation would be higher than that of a reversal.[18] Conversely, if the final state were [00], thus forcing even small traders to walk up the book in search of liquidity, then the probability of market orders to buy would decrease for a small trader too. This is evident by looking at Table III and noticing that for the LOB&DP case the probability of a market buy order further decreases to .2142.

Table VII also shows that when depth decreases on one side of the book, e.g. from [20] to [10], trading volume decreases as both market orders to sell and market orders to buy decrease. However, one should notice that the decrease in volume is more contained for the LOB&DP framework. The total percentage decrease in market sell and buy orders is equal to .1945 in the LOB, whereas it is only .1559 in the presence of a DP. When depth decreases on the ask side of the book, competition for large limit sell orders increases as large traders move from the DP to the book: this means that when the book needs liquidity to attract market orders, this is drained from the DP, which functions like a liquidity buffer. This evidence is reminiscent of Buti, Rindi and Werner (2011) who find that DP volume decreases significantly in relative order imbalances.

Proposition 5 offers at least two empirical implications for the dynamic pattern of the order flow: first, the model predicts that liquid stocks should exhibit a probability of a continuation which is higher than that of a reversal, whereas for illiquid stocks the opposite should hold; second, the model foresees an externality originating from the coexistence of a limit order book with a DP. When market depth on the former decreases (increases), it creates a liquidity injection (drain) from the DP to the limit order book.

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[18]For brevity we do not report these results here.

V   Asymmetric Information on the State of the DP

The Security and Exchange Commission has recently proposed various changes in the regulation of non-public trading interest that have been grouped under the SEC release No. 34-60997. IOI messages create a leakage of privileged information to only some select investors with access to DP. Hence, this proposal aims at enhancing DP transparency by leveling the playing field. In this Section, we extend the model and include asymmetric information on the state of the DP to illustrate the effects on market quality of a stylized two-tiered market where some traders get a preview of DP liquidity.

Assume that, all else equal, one group of large traders receives IOI or Alert messages, such as Flash orders, about the state of the DP. This feature can be embedded in the model by assuming that at each trading round nature selects with probability 1/2 a small trader, with probability 1/4 either a large uninformed trader or a large informed one. If a trader arrives at the market and is informed, then he knows the state of the DP and trades accordingly. In the spirit of the SEC proposal, we also discuss the case where the state of the DP is visible to all large traders. The following Proposition summarizes the results obtained for this two-tiered market.

Proposition 6 *When some large traders receive private information on the state of the DP,*

- *the probability that large traders, whether informed or uninformed, choose to trade in the DP increases and hence orders move from the LOB to the DP;*

- *the quality of the LOB measured by depth and best bid-offer improves, trading volume in the LOB decreases, whereas total volume increases.*

When all large traders are informed about the state of the DP, these effects become stronger.

Panels A and B of Table VIII summarize the results obtained in this extended version of the model. Panel A reports the equilibrium trading strategies of large informed and uninformed traders, and Panel B reports those of small traders. The model has been solved by starting at $T-2$ with 2 shares on both sides of the LOB. This is the regime with greater access to the DP and hence it is the most interesting to discuss the role of informed messages. By comparing the results for the LOB&DP&IOI protocol at $T-2$ with those from the protocol with only one type of large trader, it can be noticed that when information on the state of the DP is asymmetric, on average large traders use the DP more frequently (.1078). As large informed traders observe the state of the DP, they will use it very intensively when it is full on one side (.5034), and this more than compensates the tiny probability with which they use the DP when it is empty (.0123). If instead the trader who arrives is a large uninformed one, he submits an order to the DP with probability equal to .0437, which means that, compared to the case with only one type of large traders (.0279), he also uses the DP more intensively. This is due to the fact that at $T-2$ he anticipates that large informed traders will submit their orders to the DP more frequently, and that, for this reason, the DP volume will be enhanced, with the result that the execution probability of the orders submitted to the DP will increase. And if at $T-2$ he does not observe any trade (Table VIII, Panel A.2), the probability that at $T-1$ he submits to the DP increases even further (.052) than in the case without informed messages, as he knows that the probability of DP trading is higher under asymmetric information.

[**Insert Table VIII here**]

It follows that if IOI and Alert messages create a two-tiered market, with some large traders holding precise information about the state of the DP, then liquidity moves from the LOB to the DP. In fact all traders anticipate that the informed will use the DP more

extensively and this increases the probability of execution of DP orders thus reinforcing the DP externality effect.

Table IX shows that when traders are more likely to use the DP, spreads and depth in the LOB improve. Compared to the protocol without asymmetric information, here not only the spread improves but also market depth increases; the reason being that Alert and IOI messages have the overall effect of reducing the execution risk of DP trading, thus making market orders less attractive than DP orders. Considering again the probability of order submission under asymmetric information, Table VIII (Panel A.1) shows that with IOI and Alert messages the probability to observe market orders decreases by 18.1% compared to the LOB benchmark, whereas without asymmetric information the reduction is tiny (1.6%). Further, limit order submissions decrease less with asymmetric information, even though the difference in difference is much smaller. The result of this change in order submission probabilities is that volume in the LOB decreases even more than in the case without information leakage. However, due to the heavier use of the DP, total volume executed in both the LOB and the DP increases to 4.1727 (Table IX).

[**Insert Table IX here**]

Notice that large traders resort to the DP even more intensively when they can observe its content, which amplifies all the effects on market quality previously illustrated. Indeed, the order matching in the DP is enhanced by visibility and so is the attractiveness of this trading venue. Even if these results seem to support the recent SEC proposal in favour of greater DP pre-trade transparency, they should be interpreted with caution. For example, the benefit of DP trading could be jeopardized by traders taking advantage of the DP imbalance visibility to strategically manipulate prices on the LOB.

In conclusion, the result of introducing Flash orders is to reduce execution risk from DP trading. This induces those traders who are sensitive to this type of risk to switch from market to DP orders. With less market orders but more trading in the DP, LOB volume decreases, but LOB depth as well as total volume increase.

VI   Empirical and Policy Implications

Our model generates a rich set of empirical predictions. First of all, our results show that when a DP is added to a LOB, volume migrate to the DP so that volume in the LOB decreases; yet, the sum of the volume traded on both the LOB and the DP increases. Second, we show that the overall effect of intermarket competition crucially depends on how deep and tight the LOB is. We expect trade migration to be more intense when the book is deep than when it is shallow as in the latter competition from limit orders crowds out DP orders. Following the volume migration, depth at the top of the book deteriorates, whereas the effect on inside spread depends on the state of the book. When the book is very deep, the relative proportion of market to limit orders that move to the DP leaves the inside spread very tight, whereas when the book is shallow, it widens the spread. Our results also show that when traders believe that liquidity is growing in the dark pool, dark trading is enhanced so that we expect DP volume in tight books to increase more intensively than in shallow ones.

Beside depth and spread, our model has also suggestions for a third determinant of DP trading, as it shows that DP volume increases with the tick size. An increase in the tick size increases the inside spread, thus making market orders more expensive and limit orders more attractive. As a result, relatively impatient traders will switch to DP orders. This empirical prediction should be tested with caution. The reason is that the type of DP that our model

features is either independent crossing networks or DP operated within Exchanges, and thus differ from those internalization pools that are used by broker-dealers to internalize trades. Also in a framework with competition between a LOB and an internalization pool an increase of the tick size raises dark volume, but the effect is driven by the increase in broker-dealers' profits from sub-penny trading (see Buti et al., 2011). Consequently, to separate the effect that a tick size change can have on different dark venues, empiricists should control for the average order size that in internalization pools is much smaller than in traditional dark pools (*Rosenblatt Securities*, February 2011).

Our benchmark model also qualifies standard results on systematic pattern of LOB order flows, as it shows that Parlour's (1998) main finding that the probability of a continuation is larger than that of a reversal only characterizes stocks with large depth available, the opposite holding for stock with low depth. This is a ready testable empirical implication that can be addressed both over time and across different stocks: as the book becomes deeper, the probability of observing trades of the same sign should increase, the opposite taking place when the book turns shallow. Analogously, stocks with greater average depth at the BBO should be characterized by a higher probability of continuation than stocks with smaller depth at the top of the book. Furthermore, the model predicts an externality originating from the coexistence of a LOB and a DP, as it shows that when market depth decreases on one side of the LOB, thus generating order imbalance, it creates a liquidity injection from the DP into the LOB. This prediction can be tested empirically by investigating the correlation between the book imbalance and traders' DP usage.

The model's results also allow us to comment on the recent SEC proposal, aimed at increasing DP pre-trade transparency and levelling the playing field when Flash orders are used to send indications of interest to select investors. We show that by allowing traders to

observe the DP imbalance via IOI messages, LOB depth and spread, as well as total volume, improve. Furthermore, the use of IOI messages tends to move volume from the LOB to the DP and hence to reinforce the liquidity-externality effect. We can therefore conclude that the increase of DP transparency associated with a wider use of IOI messages, benefits all traders who have access to the DP including those who do not directly receive indications of interest. However, our model also shows that in general when a DP is added to a LOB, depth and volume decrease. This means that traders who are not allowed to access the DP, as it is the case for retail traders, might be harmed by the existence of this facility. Hence our model provides a motivation for the even more recent issue raised by the SEC regarding retail traders' access to dark markets. An interesting extension of our model would be to embed access to the DP for small traders. All else equal, we expect that this should strengthen our results. Moreover, it could also pave the way for high frequency and algo trading to enter DP and take advantage of the institutional order flows. The SEC should therefore carefully balance the advantages and disadvantages that a widespread access to DP would entail.

VII   Conclusions

The dynamic microstructure model presented in this paper solves for the equilibrium trading strategies of different agents who can choose to trade either in a Limit Order Book (LOB) or in a Dark Pool (DP). A DP is an Alternative Trading System that does not provide its best-priced orders for inclusion in the consolidated quotation data. The existing theory shows that dark crossing networks increase liquidity. Conversely our model shows that this is true only when a DP is added to a dealership market where traders cannot compete for the provision of liquidity by submitting limit orders. Indeed, when a DP is added to a LOB,

orders migrate away from the LOB to the dark market. The model thus demonstrates that the dark option offered to market participants produces order migration rather than order creation.

We also show that current DP orders stimulate the arrival of future DP orders thus increasing their execution probability (liquidity-externality effect). Traders' choice between LOB and DP depends both on the current state of the LOB and on the agents' expectations on the state of the DP. The model shows that high depth and small spread increase traders' use of DP, and that a reduction in the tick size makes market orders less expensive thus crowding out DP orders.

In terms of market quality, when a DP is added to a LOB we find that depth and volume deteriorate on the latter, whilst total volume increases. The effect of the introduction of a DP on the inside spread depends on the state of the book, improving when it is deep and worsening when it is shallow.

The model also offers new insights on the systematic patterns of order flow that can arise from traders' interaction with the LOB. Specifically, when the book is deep, the probability of a continuation is greater than that of a reversal, the opposite being true when the book is shallow. Furthermore, DP act as liquidity buffers by supplying liquidity after a reduction of market depth on the LOB.

Finally, we show that when some traders are allowed to observe the state of the DP via Flash orders, order migration from the LOB to the DP increases. The reason is that when traders know that other traders are informed on the state of the DP, they anticipate that the informed will use the DP more intensively and that this will increase the execution probability of DP orders. However, we also show that the use of Flash orders can improve both spread and depth, as well as total volume.

Our model focuses on the competition between a transparent LOB and a dark market. However today's regulated exchanges allow traders to opt also for undisclosed orders, thus offering an alternative to DP trading. Among the wide range of existing undisclosed orders, the closest competitors to DP orders are Hidden Mid-Point Peg which are totally invisible and are submitted at the spread mid-point. Compared to DP orders, Hidden Mid-Point Peg on the one hand can be executed against the LOB order flow and therefore they can have a higher execution probability; on the other hand, by standing on a public LOB, they can be more easily detected by traders in search of hidden liquidity. Tackling the issue of competition on dark liquidity between regulated exchanges and Alternative Trading Systems is therefore a thoroughly interesting issue that we leave for future research.

Appendix

Proof of Proposition 1

Consider first the benchmark case. The model is solved by backward induction, starting from $t = T$. The T-trader solves a simplified version of program (2), if large, or (3), if small:

$$\max_{\varphi} \pi_T^e \left\{ \varphi(j, \overline{p}_i^B), \varphi(2, \overline{p}^B), \varphi(0), \varphi(j, \overline{p}_i^A), \varphi(2, \overline{p}^A) | \beta_T, b_T \right\} \tag{2'}$$

$$\max_{\varphi} \pi_T^e \left\{ \varphi(1, \overline{p}_i^B), \varphi(0), \varphi(1, \overline{p}_i^A) | \beta_T, b_T \right\} \tag{3'}$$

Without loss of generality, assume that depending on β_T and the state of the book b_T the trader selects one equilibrium strategy φ_a^n, with $a = \{ST, LT\}$ and $n \in N_T$, being N_T the number of the equilibrium strategies at T. The β-thresholds between two different strategies are determined as follows:

$$\beta_T^{\varphi_a^{n-1}, \varphi_a^n} : \pi_T^e(\varphi_a^{n-1} \mid \Omega_T) - \pi_T^e(\varphi_a^n \mid \Omega_T) = 0$$

Notice that these strategies are ordered in such a way that the β-thresholds are increasing, $\beta_T^{\varphi_a^{n-1}, \varphi_a^n} < \beta_T^{\varphi_a^n, \varphi_a^{n+1}}$. Hence, the ex-ante probability that a trader submits a certain order type at T is determined as follows:

$$\Pr_T(\varphi_a^n \mid \Omega_T) = F(\beta_T^{\varphi_a^n, \varphi_a^{n-1}} | \Omega_T) - F(\beta_T^{\varphi_a^{n-1}, \varphi_a^n} | \Omega_T)$$

Consider now period $t = T - 1$. The incoming trader solves program (2) or (3) if large or small respectively, and uses $\Pr_T(\varphi_a^n \mid \Omega_T)$ to compute the execution probabilities of his limit orders. Given the optimal strategies at T, the β-thresholds and the order type probabilities at $T - 1$ are derived using the same procedure as for period T, which is then reiterated for period $T - 2$. The solution of the LOB&DP and DM&DP models follow the same methodology, but now the large trader solves program (4) or (5) respectively.

We provide examples for the LOB&DP protocol for the three trading periods analyzed; these examples belong to the case where the book opens as $b_{T-2} = [22]$. From now onwards we assume that for large traders the optimal order size is $j^* = \max_j [\varphi \mid \Omega_t]$, since $\partial \pi_t^e(\varphi)/\partial j \geq 0$ due to agents' risk neutrality.

Consider the following books at T: (a) $b_T = [20]$, $vis_{T-2} \; vis_{T-1}$, (b) $b_T = [20]$, $inv_{T-2} \; vis_{T-1}$. In the first case traders observe a change in the LOB in both periods, while in the second one only at $T - 1$. We focus on the large trader's profits that for (a) are:

$$\pi_T^e[\varphi(2, \overline{p}_2^B) \mid \Omega_T^{[20, vv]}] = 2(p_2^B - \beta_T v) = 2(1 - \tfrac{3\tau}{2} - \beta_T)$$

$$\pi_T^e[\varphi(2,\overline{p}_1^A) \mid \Omega_T^{[20,vv]}] = 2(\beta_T - v p_1^A) = 2(\beta_T - 1 - \tfrac{\tau}{2})$$

$$\pi_T^e[\varphi(-2,p_{Mid}) \mid \Omega_T^{[20,vv]}] = 2(\tfrac{p_1^A+p_2^B}{2} - \beta_T v)\Pr_{-2}(\tfrac{p_1^A+p_2^B}{2}|\Omega_T) = 2(1-\tfrac{\tau}{2}-\beta_T)\times\tfrac{1}{3}$$

$$\pi_T^e[\varphi(+2,p_{Mid}) \mid \Omega_T^{[20,vv]}] = 2(\beta_T v - \tfrac{p_1^A+p_2^B}{2})\Pr_{+2}(\tfrac{p_1^A+p_2^B}{2}|\Omega_T) = 2(\beta_T - 1+\tfrac{\tau}{2})\times\tfrac{1}{3}$$

where $\Omega_T^{[b_T,y_{T-2}y_{T-1}]}$ and $y_t \in \{v,i\}$, with $v = vis_t$ and $i = inv_t$. By solving program (2') for this case it is straightforward to show that all strategies are optimal in equilibrium ($N_T = 4$) and that for the LT: $\varphi_{LT,[20,vv]}^1 = \varphi(2,\overline{p}_2^B)$, $\varphi_{LT,[20,vv]}^2 = \varphi(-2,p_{Mid})$, $\varphi_{LT,[20,vv]}^3 = \varphi(+2,p_{Mid})$ and $\varphi_{LT,[20,vv]}^4 = \varphi(2,\overline{p}_1^A)$. As an example we compute the probability of $\varphi_{LT,[20,vv]}^1$ and to ease the notation from now onwards we omit the subscript "LT":

$$\beta_T^{\varphi_{[20,vv]}^1,\varphi_{[20,vv]}^2} \quad : \quad \pi_T^e[\varphi_{[20,vv]}^1] - \pi_T^e[\varphi_{[20,vv]}^2] = 0 \rightarrow \beta_T^{\varphi_{[20,vv]}^1,\varphi_{[20,vv]}^2} = 1 - 2\tau$$

$$\Pr_T(\varphi_{[20,vv]}^1) = F(\beta_T^{\varphi_{[20,vv]}^1,\varphi_{[20,vv]}^2}) = \tfrac{1}{2}(1-2\tau)$$

In case (b), profits for DP orders differ:

$$\pi_T^e[\varphi(-2,p_{Mid}) \mid \Omega_T^{[20,iv]}] = 2(\tfrac{p_1^A+p_2^B}{2}-\beta_T v)(\tfrac{1}{3}\times 1+\tfrac{1}{3}\tfrac{\Pr_{T-2}(\varphi^n(+2,p_{Mid}))}{\Pr_{T-2}(\varphi^n(+2,p_{Mid}))+\Pr_{T-2}(\varphi^n(-2,p_{Mid}))+\Pr_{T-2}(\varphi^n(0))})$$

$$\pi_T^e[\varphi(+2,p_{Mid}) \mid \Omega_T^{[20,iv]}] = 2(\beta_T v - \tfrac{p_1^A+p_2^B}{2})(\tfrac{1}{3}\times 1+\tfrac{1}{3}\tfrac{\Pr_{T-2}(\varphi^n(-2,p_{Mid}))}{\Pr_{T-2}(\varphi^n(+2,p_{Mid}))+\Pr_{T-2}(\varphi^n(-2,p_{Mid}))+\Pr_{T-2}(\varphi^n(0))})$$

where $\varphi^n(..)$ are equilibrium strategies, and we omit that all probabilities at $T-2$ are conditional to Ω_{T-2}. In this case both the β-thresholds and the order probabilities are a function of the equilibrium order strategies at $T-2$, that are rationally computed by the T-trader. For example, if the equilibrium strategies are such that $\varphi_{[20,iv]}^1 = \varphi(2,\overline{p}_2^B)$ and $\varphi_{[20,iv]}^2 = \varphi(-2,p_{Mid})$, we obtain:

$$\beta_T^{\varphi_{[20,iv]}^1,\varphi_{[20,iv]}^2} \quad : \quad \pi_T^e[\varphi_{[20,iv]}^1] - \pi_T^e[\varphi_{[20,iv]}^2] = 0$$

$$\beta_T^{\varphi_{[20,iv]}^1,\varphi_{[20,iv]}^2} = \tfrac{(2-7\tau)\Pr_{T-2}(\varphi^n(+2,p_{Mid}))+4(1-2\tau)[\Pr_{T-2}(\varphi^n(-2,p_{Mid}))+\Pr_{T-2}(\varphi^n(0))]}{2\Pr_{T-2}(\varphi^n(+2,p_{Mid}))+4[\Pr_{T-2}(\varphi^n(-2,p_{Mid}))+\Pr_{T-2}(\varphi^n(0))]}$$

$$\Pr_T(\varphi_{[20,iv]}^1) = F(\beta_T^{\varphi_{[20,iv]}^1,\varphi_{[20,iv]}^2}) = \tfrac{1}{2}\beta_T^{\varphi_{[20,iv]}^1,\varphi_{[20,iv]}^2}$$

To determine the equilibrium strategies $\varphi_{[20,iv]}^n$ at T for $n \in N_T$, the model has to be solved up to period $T-2$. We anticipate that $\varphi(2,\overline{p}_2^B)$ is indeed an equilibrium strategy, and that the corresponding probability is: $\Pr_T(\varphi_{[20,iv]}^1) = \tfrac{(2-5\tau)}{4}$.

For $T-1$ and $T-2$ we only specify the profit formulas, as the derivation of the β-thresholds

and order probabilities follow the same steps presented for period T. Consider the case $b_{T-1} = [20]$, vis_{T-2}. Small traders' profits are as follows:

$$\pi^e_{T-1}[\varphi(1, \overline{p}^B_2) \mid \Omega^{[20,v]}_{T-1}] = (p^B_2 - \beta_{T-1}v)$$

$$\pi^e_{T-1}[\varphi(1, p^A_1) \mid \Omega^{[20,v]}_{T-1}] = \pi^e_{T-1}[\varphi(0)] = 0$$

$$\pi^e_{T-1}[\varphi(1, p^B_1) \mid \Omega^{[20,v]}_{T-1}] = (\beta_{T-1}v - p^B_1)\tfrac{1}{2}[\Pr_T(\varphi(1, \overline{p}^B_1) \mid \Omega^{[21,vv]}_T) + \Pr_T(\varphi(2, \overline{p}^B_1) \mid \Omega^{[21,vv]}_T)]$$

$$\pi^e_{T-1}[\varphi(1, \overline{p}^A_1) \mid \Omega^{[20,v]}_{T-1}] = (\beta_{T-1}v - p^A_1)$$

where here we condition to $\Omega^{[b_{T-1}, y_{T-2}]}_{T-1}$. Large traders' strategies are similar, the only difference being that $j = 2$, and that they can submit DP orders:

$$\pi^e_{T-1}[\varphi(-2, p_{Mid})] = E[(p_{Mid} - \beta_{T-1}v) \Pr_{-2}(p_{Mid} \mid \Omega_T)]$$

$$\pi^e_{T-1}[\varphi(+2, p_{Mid})] = E[(\beta_{T-1}v - p_{Mid}) \Pr_{+2}(p_{Mid} \mid \Omega_T)]$$

We specify the first one:

$$\pi^e_{T-1}[\varphi(-2, p_{Mid})] = \tfrac{1}{3} \times 2 \times \tfrac{1}{2}(\tfrac{p^A_1 + p^B_2}{2} - \beta_{T-1}v)\Pr_T(\varphi(+2, p_{Mid}) \mid \Omega^{[20,vi,0]}_T)$$

$$+ \tfrac{1}{3} \times 2 \times \tfrac{1}{2}\{(\tfrac{p^A_2 + p^B_2}{2} - \beta_{T-1}v)\Pr_T(\varphi(2, \overline{p}^A_1) \mid \Omega^{[20,vi,+6]}_T) + (\tfrac{p^A_1 + p^B_2}{2} - \beta_{T-1}v)$$

$$[1 + \Pr_T(\varphi(+2, p_{Mid}) \mid \Omega^{[20,vi,+6]}_T) + \Pr_T(\varphi(-2, p_{Mid}) \mid \Omega^{[20,vi,+6]}_T) + \Pr_T(\varphi(2, \overline{p}^B_2) \mid \Omega^{[20,vi,+6]}_T)]\}$$

where now we also condition on the state of the DP, $\Omega^{[b_T, y_{T-2}y_{T-1}, DP_T]}_T$.

At $T - 2$ we consider the book $b_{T-2} = [22]$ and present profit formulas only for the sell side of the market, the buy side being symmetric:

$$\pi^e_{T-2}[\varphi(2, \overline{p}^B_1)] = 2(p^B_1 - \beta_{T-2}v)$$

$$\pi^e_{T-2}[\varphi(0)] = 0$$

$$\pi^e_{T-2}[\varphi(2, p^A_1)] = (p^A_1 - \beta_{T-2}v)\{\tfrac{1}{2}\Pr_{T-1}(\varphi(1, \overline{p}^A_1) \mid \Omega^{[42,v]}_{T-1})[\tfrac{1}{2}\Pr_T(\varphi(2, \overline{p}^A_1) \mid \Omega^{[32,vv]}_T)]$$

$$+ \tfrac{1}{2}\Pr_{T-1}(\varphi(2, \overline{p}^A_1) \mid \Omega^{[42,v]}_{T-1})[\tfrac{1}{2}\Pr_T(\varphi(1, \overline{p}^A_1) \mid \Omega^{[22,vv]}_T) + \tfrac{1}{2}2\Pr_T(\varphi(2, \overline{p}^A_1) \mid \Omega^{[22,vv]}_T)]\}$$

$$\pi^e_{T-2}[\varphi(-2, p_{Mid})] = E[(p_{Mid} - \beta_{T-2}v) \Pr_{-2}(p_{Mid} \mid \Omega_T)]$$

where to economize space we do not specify the formula for $\pi^e_{T-2}[\varphi(-2, p_{Mid})]$. Results from Proposition 1 are derived by comparing equilibrium strategies for the three cases: LOB, LOB&DP and DM&DP, presented in Table I and Table II. In Figures A1-A3 we provide plots at $T - 2$ for the large trader's profits as a function of β, whereas in Figure A4 we focus

on $T-1$. Notice that each Figure relates to a different point of Proposition 1 and provides a graphical representation of the traders' optimization problem. For example, Figure A1 shows how the introduction of DP orders changes the optimal order submission strategies of large traders by crowding out both market and limit orders. We consider only selling strategies, the plots being symmetric for the buy side.
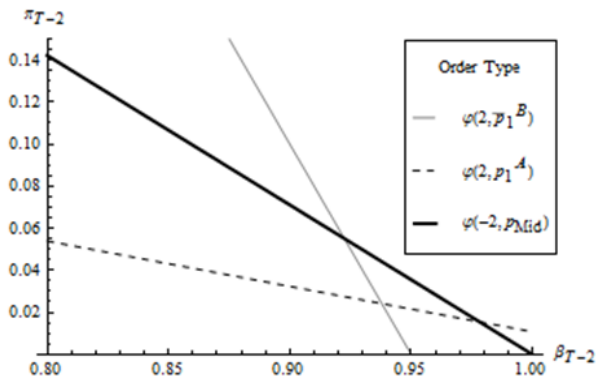

Figure A1. Trade Migration on the LOB&DP $-$ b$_{T-2}$=[22]
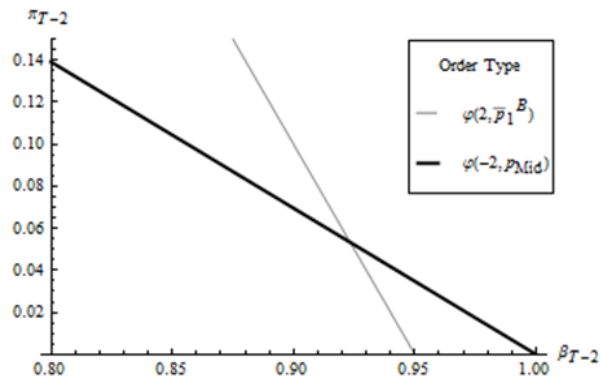
Figure A2. Trade Creation on the DM&DP
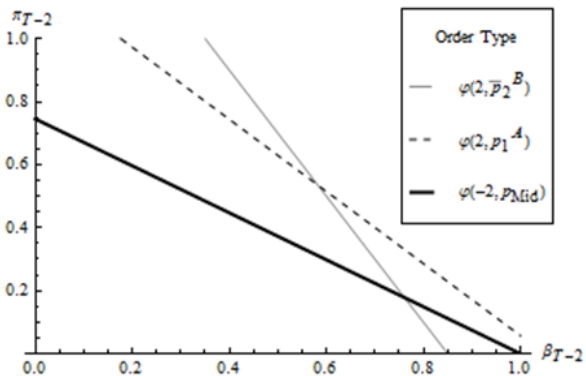
Figure A3. Trade Migration on the LOB&DP $-$ b$_{T-2}$=[00]
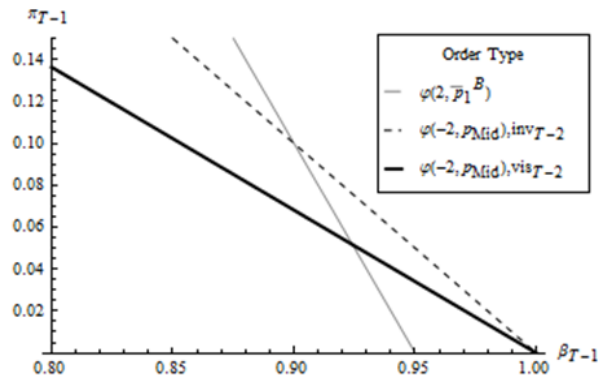
Figure A4. Liquidity Externality Effect of DP $-$ b$_{T-1}$=[22]

Proof of Proposition 2

Results from Proposition 2 are obtained by straightforward comparison of the equilibrium strategies derived in the proof of Proposition 1 for different states of the LOB. We provide graphical plots also for this proof. Consider first the DP&LOB: compare Figures A1 and A5 for the effect of market depth, A1 and A3 for the effect of spread, and A5 and A6 for the tick size. For DM&DP, consider Figure A7.

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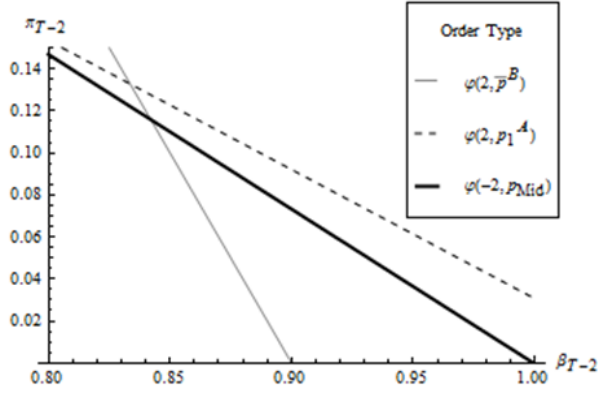
Figure A5. Dark Pool Drivers – LOB&DP – $b_{T\text{-}2}$=[11]
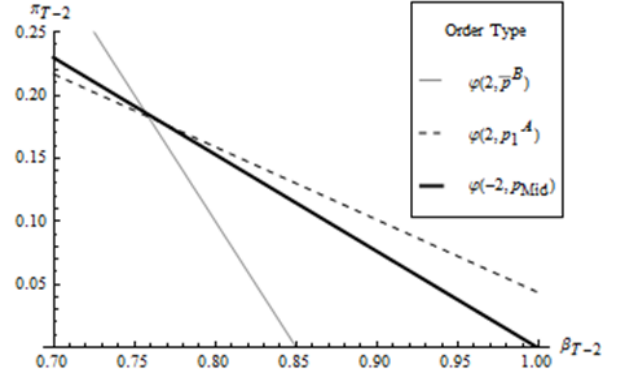
Figure A6. Dark Pool Drivers - $b_{T\text{-}2}$=[11] – τ=0.15
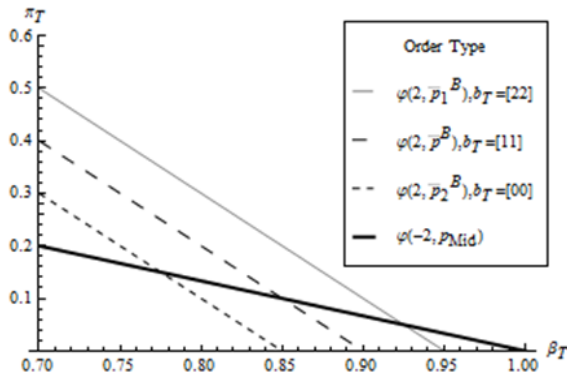
Figure A7. Dark Pool Drivers – DM&DP
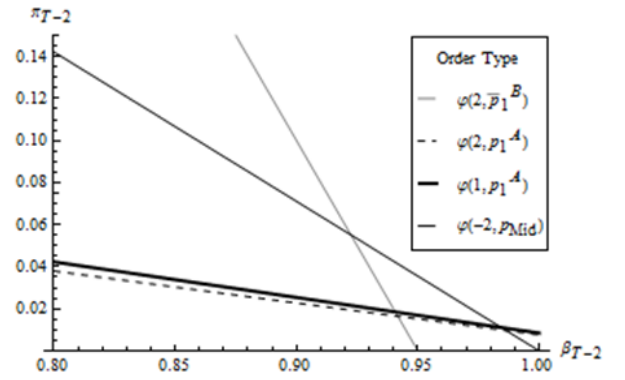
Figure A8. Price Pressure – $b_{T\text{-}2}$=[22]

Proof of Proposition 3

The benchmark and the LOB&DP model are solved following the procedure already illustrated in Proposition 1. Consider an opening book $b_{T-2} = [22]$ where a large limit sell order, $\varphi_{T-2}(2, p_1^A)$, is submitted: with no price pressure, at $T - 1$ the book is full, $b_{T-1} = [42]$, so limit orders are not equilibrium strategies for next period traders; with price pressure, instead, the book turns empty, $b_{T-1} = [00]$ and limit orders, $\varphi_{T-1}(j, p_1^A)$ and $\varphi_{T-1}(j, p_1^B)$, are included in the set of the available strategies thus allowing for undercutting of existing limit orders. Hence, as large traders at $T - 2$ rationally anticipate that price pressure will expose them to undercutting, $j = 2$ might not necessarily be optimal anymore. Indeed, in program

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(4) profits from a 2-unit limit sell order are modified as follows (buy side is symmetric):

$$\pi^e_{T-2}[\varphi(2,p^A_1)] = (p^A_1 - \beta_{T-2}v_T)\{\tfrac{1}{2}\Pr_{T-1}(\varphi(1,\overline{p}^{A_d}_2) \mid \Omega^{[00,v]}_{T-1})[\tfrac{1}{2}\Pr_T(\varphi(2,\overline{p}^A_1) \mid \Omega^{[20,vv]}_T)]$$

$$+ \tfrac{1}{2}\Pr_{T-1}(\varphi(2,\overline{p}^{A_d}_2) \mid \Omega^{[00,v]}_{T-1})[\tfrac{1}{2}\Pr_T(\varphi(1,\overline{p}^A_1) \mid \Omega^{[22,vv]}_T) + \tfrac{1}{2}2\Pr_T(\varphi(2,\overline{p}^A_1) \mid \Omega^{[20,vv]}_T)]\}$$

where A_d is the ask price after the price pressure. In Figure A8 we show that with price pressure 1-unit limit sell orders are more profitable than 2-unit ones. Comparison with Figure A1 shows that the $\beta-$interval where DP orders are optimal increases. Numerical values are reported in Table V.

Proof of Proposition 4

Results presented in Table VI, are obtained by comparing the three market quality measures for both the benchmark and the LOB&DP protocol. As an example, we consider the LOB&DP model with an opening book equal to $b_{T-2} = [22]$ -hence omitted in subscript for φ- and specify formulas for the estimated spread and depth at $T-1$ and for the executed volume at $T-2$. Similar computations make it possible to derive the market quality measures for all the other cases. We define equilibrium strategies at $T-2$ for a LT as follows: $\varphi^1_{LT} = \varphi(2,\overline{p}^B_2)$, $\varphi^2_{LT} = \varphi(-2,p_{Mid})$, $\varphi^3_{LT} = \varphi(2,p^A_1)$, $\varphi^4_{LT} = \varphi(2,p^B_1)$, $\varphi^5_{LT} = \varphi(+2,p_{Mid})$ and $\varphi^6_{LT} = \varphi(2,\overline{p}^A_1)$. The ones for a ST are: $\varphi^1_{ST} = \varphi(1,\overline{p}^B_2)$, $\varphi^2_{ST} = \varphi(1,p^A_1)$, $\varphi^3_{ST} = \varphi(1,p^B_1)$ and $\varphi^4_{ST} = \varphi(2,\overline{p}^A_1)$.

$$S^e_{T-1,[22]} = \tfrac{1}{2}\{\Pr_{T-2}(\varphi^1_{LT})\left(p^A_1 - p^B_2\right) + \Pr_{T-2}(\varphi^6_{LT})\left(p^A_2 - p^B_1\right)$$

$$+ [\Pr_{T-2}(\varphi^2_{LT}) + \Pr_{T-2}(\varphi^3_{LT}) + \Pr_{T-2}(\varphi^4_{LT}) + \Pr_{T-2}(\varphi^5_{LT})]\left(p^A_1 - p^B_1\right)\}$$

$$+ \tfrac{1}{2}[\Pr_{T-2}(\varphi^1_{ST}) + \Pr_{T-2}(\varphi^2_{ST}) + \Pr_{T-2}(\varphi^3_{ST}) + \Pr_{T-2}(\varphi^4_{ST})]\left(p^A_1 - p^B_1\right)$$

$$D^e_{T-1,[22]} = \tfrac{1}{2}\{2 \times [\Pr_{T-2}(\varphi^1_{LT}) + \Pr_{T-2}(\varphi^6_{LT})]$$

$$+ 6 \times [\Pr_{T-2}(\varphi^3_{LT}) + \Pr_{T-2}(\varphi^4_{LT})] + 4 \times [\Pr_{T-2}(\varphi^2_{LT}) + \Pr_{T-2}(\varphi^5_{LT})]\}$$

$$+ \tfrac{1}{2}\{3 \times [\Pr_{T-2}(\varphi^1_{ST}) + \Pr_{T-2}(\varphi^4_{ST})] + 5 \times [\Pr_{T-2}(\varphi^2_{ST}) + \Pr_{T-2}(\varphi^3_{ST})]\}$$

$$V^e_{T-2,[22]} = \tfrac{1}{2}[\Pr_{T-2}(\varphi^1_{ST}) + \Pr_{T-2}(\varphi^4_{ST})] + \tfrac{1}{2}\times 2[\Pr_{T-2}(\varphi^1_{LT}) + \Pr_{T-2}(\varphi^6_{LT})]$$

Proof of Proposition 5

Results are derived by comparing equilibrium trading strategies for two new starting books: $b_{T-2} = [20]$ and $b_{T-2} = [10]$. As the solutions of these two cases follow the same steps as the one presented in the proof of Proposition 1 for the book $b_{T-2} = [22]$, they are omitted and available at the authors upon request. In Figure A9 we plot the small traders' profits for the buy side and show that when the initial state of the book moves from [20] to [10], market buy orders increase. Analogously, Figure A10 shows large traders' profits and indicates that market buy orders decrease substantially. Numerical values are provided in Table VII.
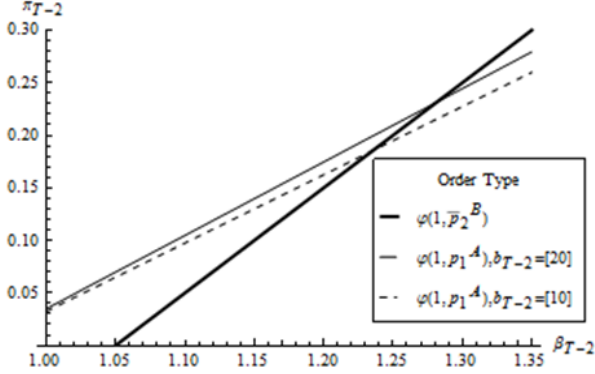
Figure A9. Pattern in Order Flow – ST
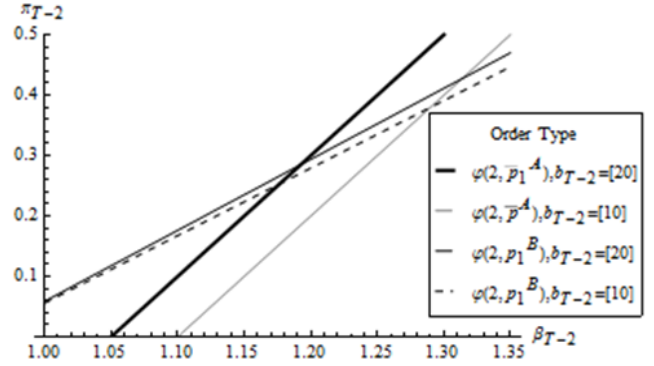
Figure A10. Pattern in Order Flow – LT

Proof of Proposition 6

This proof follows the same methodology presented in Proposition 1. To ease the comparison with the previous framework, we provide again as an example the case with $b_T = [20]$, inv_{T-2} vis_{T-1}, for both uninformed (u) and informed (i) large traders.[19] Considering first the u-trader, profits from market orders are unchanged and omitted, but profits from DP orders differ. For example profits from a DP sell order become:

$$\pi_T^e[\varphi_u(-2, p_{Mid})|\Omega_T^{[20,iv]}] = 2(\tfrac{p_1^A + p_2^B}{2} - \beta_T v)\Pr_{-2}(\tfrac{p_1^A + p_2^B}{2}|\Omega_T)$$

$$= 2(\tfrac{p_1^A + p_2^B}{2} - \beta_T v)[\tfrac{1}{3} \times 1 + \tfrac{1}{3}\sum_{d=i,u}\tfrac{\Pr_{T-2}(\varphi_d^n(-2, p_{Mid}))}{\Pr_{T-2}(\varphi_d^n(+2, p_{Mid})) + \Pr_{T-2}(\varphi_d^n(-2, p_{Mid}))}]$$

where we omit that all probabilities at $T-2$ are conditional on Ω_{T-2}. Consider now i-traders: their profits for $\varphi_i(\pm 2, p_{Mid})$ depend on the actual state of the DP that in this case can be

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[19] The case $b_T = [20]$, vis_{T-2} vis_{T-1} is not interesting as no agent has played in the DP.

$DP_T = \{\pm 2, \pm 4\}$. We provide as an example profits for $\varphi_i(-2, p_{Mid})$:

$$\pi_T^e[\varphi_i(-2, p_{Mid})|\Omega_T^{[20,iv,+2]}] = \pi_T^e[\varphi_i(-2, p_{Mid})|\Omega_T^{[20,iv,+4]}] = 2\left(\frac{p_1^A + p_2^B}{2} - \beta_T v\right)$$

$$\pi_T^e[\varphi_i(-2, p_{Mid})|\Omega_T^{[20,iv,-2]}] = \pi_T^e[\varphi_i(-2, p_{Mid})|\Omega_T^{[20,iv,-4]}] = 0$$

We omit the discussion on periods $T-1$ and $T-2$ as the intuition is similar. Profits at $T-2$ for both the u- and i-trader are presented in Figures A11 and A12-14 respectively. The case where all large traders are informed is solved similarly; profits at $T-2$ for the i-trader are presented in Figures A15 and A16. Comparison between these plots and those presented in Figure 1 shows that traders use DP orders more intensively and increasingly with the number of agents who observe the DP.
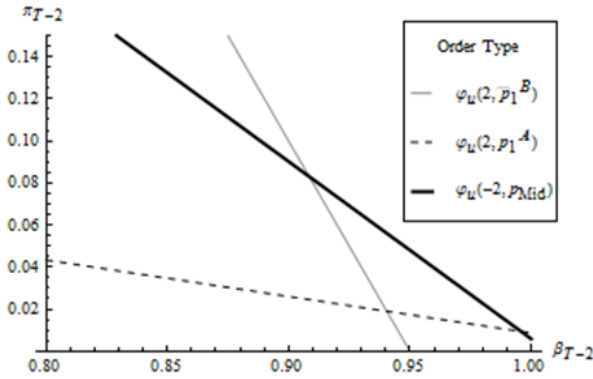
Figure A11. Uninformed LT – b_{T-2}=[22]
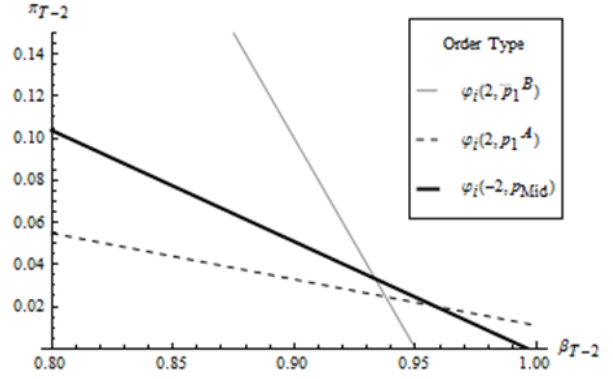
Figure A12. Informed LT – b_{T-2}=[22] - DP_{T-2}=[0]
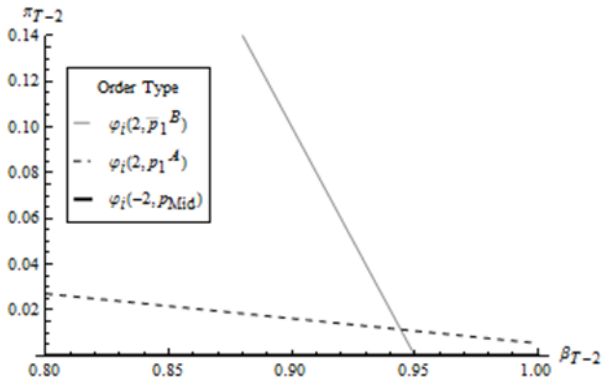
Figure A13. Informed LT – b_{T-2}=[22] - DP_{T-2}=[-6]
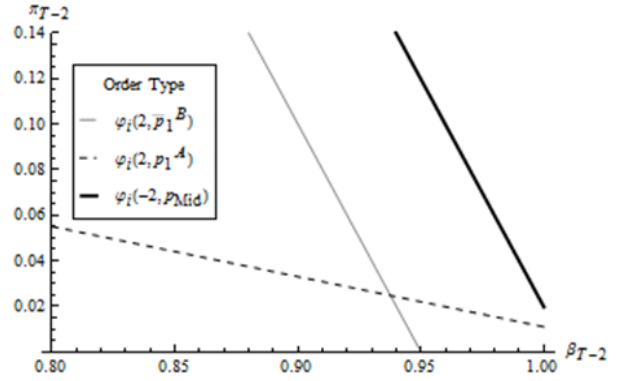
Figure A14. Informed LT – b_{T-2}=[22] - DP_{T-2}=[+6]

The first result of Proposition 6 is derived by comparing numerical values for the new equilibrium strategies with those obtained in Proposition 1. Results for market quality are obtained by using the formulas presented in the proof of Proposition 4, with the addition of i-traders. Similarly, for the case with all i-trader results are obtained by comparing the numerical values presented in Tables VIII and IX.
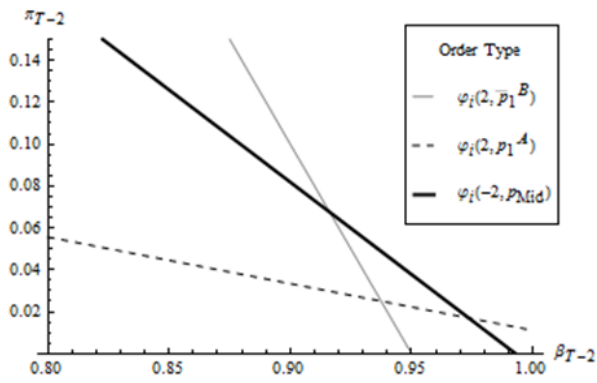
Figure A15. Only Informed LT – b_{T-2}=[22] - DP_{T-2}=[0]
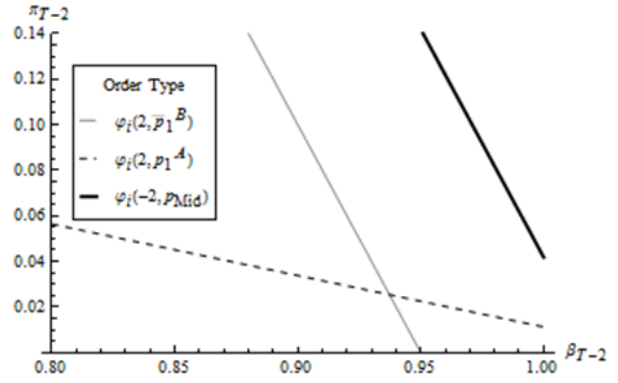
Figure A16. Only Informed LT – b_{T-2}=[22] - DP_{T-2}=[+6]

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DP volume: 12.87% (8.75%) of consolidated US equity volume, Feb11 (Feb09)

PUBLIC CROSSING NETWORKS
-Independent / Agency-

Liquidnet	0.37%
Instinet CBX	0.39%
ITG Posit	0.59%
Convergex Millennium	0.31%
Pipeline Trading	0.11%
Convergex Vortex	0.10%

CONSORTIUM-BASED POOLS

Level	0.77%
Bids	0.47%

INTERNALIZATION POOLS
-Broker Sponsored-

Knights Link	1.33%
Goldman Sigma X	1.53%
Getco Exec. Services	1.28%
Credit Suisse Crossfinder	2.82%
Citi Match	0.43%
UBS Pin	0.40%
Morgan Stanley MS Pool	0.74%
Barclays LX	0.84%

EXCHANGE-BASED POOLS

Direct Edge	0.19%
EDGX MPM	0.20%

9.37%
(Feb09: 5.02%)

1.24%
(Feb09: 0.62%)

1.87%
(Feb09: 1.09%)

0.39%
(Feb09:1.12%)

Rosenblatt Securities Inc. Feb. 2011 (2009)

Figure 1 - Dark Pools Volume: Percentage of Consolidated US Equity Volume, February 2011 and 2009.
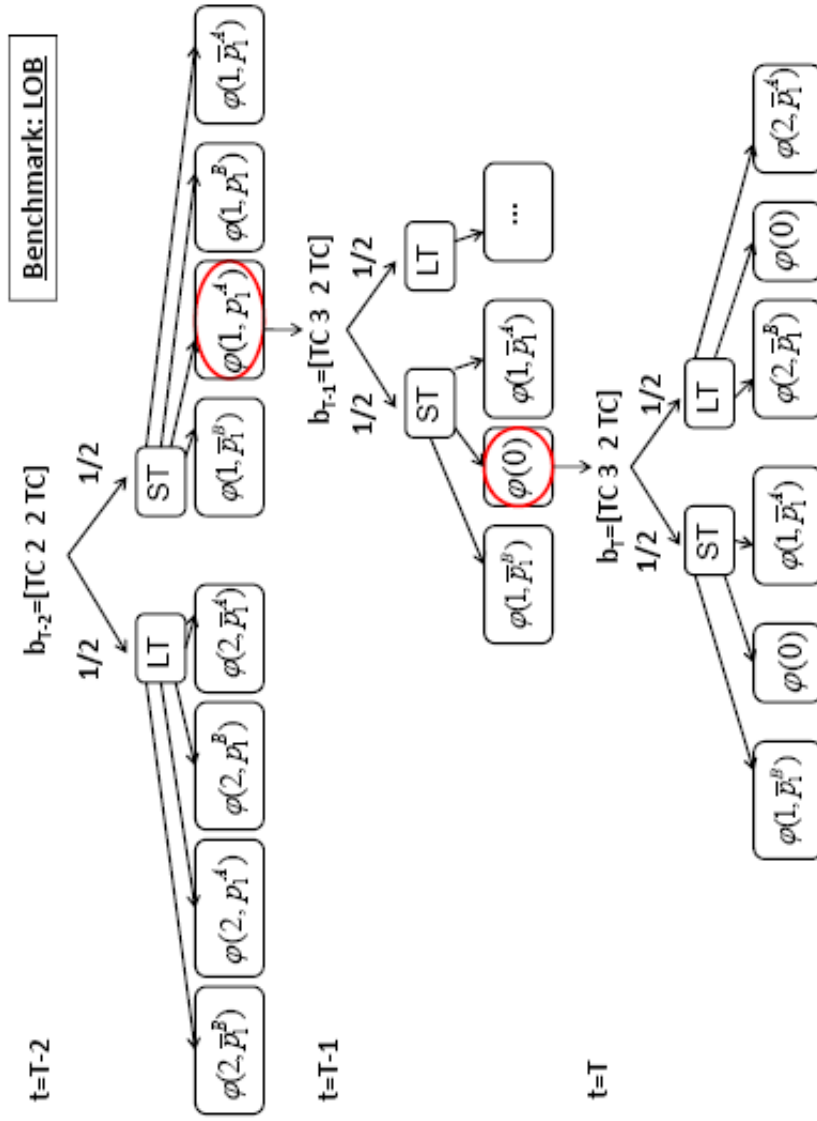
Figure 2 - Benchmark Model of Limit Order Book (LOB). Example for $j = 2$ of the extensive form of the game. The trading crowd is indicated by TC, while small and large traders are named ST and LT respectively. Only equilibrium strategies are presented.
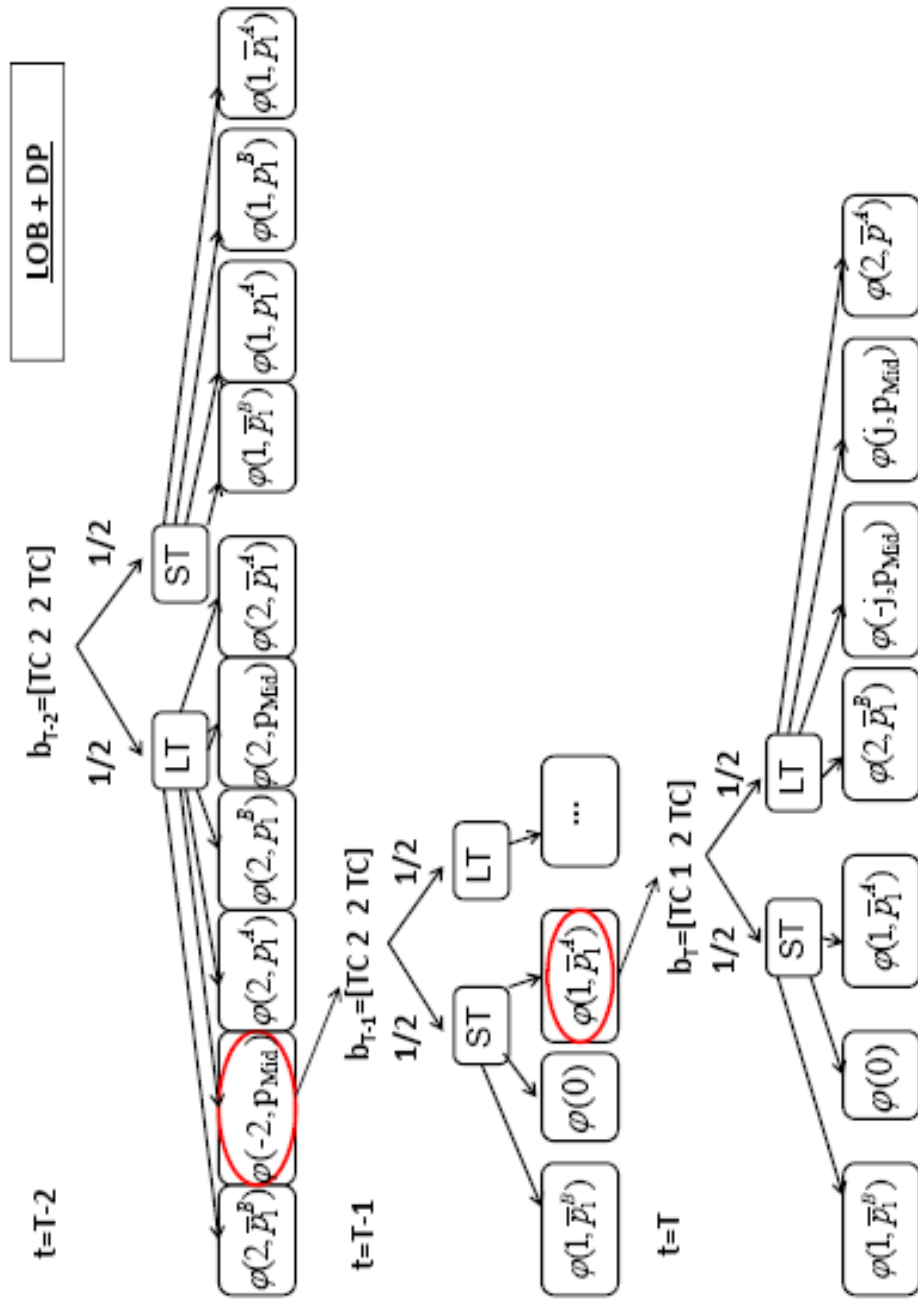
Figure 3 - Limit Order Book and Dark Pool Market (LOB&DP). Example of the extensive form of the game for $j = 2$. The trading crowd is indicated by TC, while small and large traders are named ST and LT respectively. Only equilibrium strategies are presented.

Figure 4 - Dealership Market with a Dark Pool (DM&DP). Example of the extensive form of the game for $j = 2$. The trading crowd is indicated by TC, while small and large traders are named ST and LT respectively. Only equilibrium strategies are presented.

	T-2	T-1	T
p_2^A			p_2^A
p_1^A	$\varphi(2, p_1^A)$	p_2^A	p_1^A
p_1^B		p_1^A	p_1^B
p_2^B		p_1^B	p_2^B
		p_2^B	

v

$v^d = v - \tau$

Figure 5 - LOB Mechanics. Example for the case where a 2 shares limit order to sell is submitted at $T-2$ that generates a temporary 1-tick negative price pressure.

Table I - Order Submission Probabilities - $b_{T-2,T-1,T} = [22]$

Large Trader	$T-2$			$T-1$, vis_{T-2} (inv_{T-2})			T, vis_{T-2} vis_{T-1} $(inv_{T-2}$ $vis_{T-1})$		
	LOB	LOB&DP	DM&DP	LOB	LOB&DP	DM&DP	LOB	LOB&DP	DM&DP
$\varphi(2,\overline{p}_1^B)$	.4686	.4612	.4617	.4750	.4621 (.4500)	.4621 (.4556)	.4750	.4625 (.4500)	.4625 (.4558)
$\varphi(2,p_1^A)$	.0314	.0109							
$\varphi(-j,p_{Mid})$		.0279	.0383		.0379 (.0500)	.0379 (.0444)		.0375 (.0500)	.0375 (.0442)
$\varphi(0)$				.0500			.0500		
$\varphi(+j,p_{Mid})$		.0279	.0383		.0379 (.0500)	.0379 (.0444)		.0375 (.0500)	.0375 (.0442)
$\varphi(2,p_1^B)$	.0314	.0109							
$\varphi(2,\overline{p}_1^A)$	.4686	.4612	.4617	.4750	.4621 (.4500)	.4621 (.4556)	.4750	.4625 (.4500)	.4625 (.4558)

Table I: Order Submission Probabilities - $b_{T-2,T-1,T} = [22]$. This Table reports large traders' submission probabilities for the orders listed in column 1 for the benchmark (LOB), for the LOB&DP and for the DM&DP framework respectively. Execution probabilities are reported for the state of the order book with 2 shares on both sides of the order book and for all periods T, $T-1$ and $T-2$. For example, large agents arriving at the market at $T-1$ after having observed a change in the order book at $T-2$ (vis_{T-2}), submit market orders to sell (to buy) at B_1 (A_1) with probability .4750 if they trade in the LOB, and with probability .4621 if they trade either in the LOB&DP or in the DM&DP protocol. If instead they do not observe any change in the order book at time $T-2$ (inv_{T-2}), the probabilities are .4500 for LOB&DP and .4556 for DM&DP.

Table II - Order Submission Probabilities - $b_{T-2,T-1,T} = [00]$

Large Trader	$T-2$			$T-1$, vis_{T-2} (inv_{T-2})			T, vis_{T-2} vis_{T-1} (inv_{T-2} vis_{T-1})		
	LOB	LOB&DP	DM&DP	LOB	LOB&DP	DM&DP	LOB	LOB&DP	DM&DP
$\varphi(2,\bar{p}_2^B)$	.2857	.2912		.3697	.3726 (.3500)		.4250	.3875 (.3500)	
$\varphi(2,\bar{p}_1^B)$			.4617	.1303	.1274 (.0491)	.4621 (.4556)			.4625 (.4558)
$\varphi(2,p_1^A)$	.2143	.2088			(.1009)	.0379 (.0444)		.1125 (.1500)	.0375 (.0442)
$\varphi(-2,p_{Mid})$			.0383						
$\varphi(0)$							.1500		
$\varphi(+2,p_{Mid})$			.0383						
$\varphi(2,p_1^B)$	.2143	.2088			(.1009)	.0379 (.0444)		.1125 (.1500)	.0375 (.0442)
$\varphi(2,\bar{p}_1^A)$			.4617	.1303	.1274 (.0491)	.4621 (.4556)			.4625 (.4558)
$\varphi(2,\bar{p}_2^A)$	.2857	.2912		.3697	.3726 (.3500)		.4250	.3875 (.3500)	

Table II: Order Submission Probabilities - $b_{T-2,T-1,T} = [00]$. This Table reports large traders' submission probabilities for the orders listed in column 1 for the benchmark (LOB), for the LOB&DP and for the DM&DP framework respectively. Execution probabilities are reported for the state of the limit order book with no shares on both sides and for all periods T, $T-1$ and $T-2$. For example, large agents arriving at the market at T after having observed a change in the order book both at $T-2$ and at $T-1$ (vis_{T-2} vis_{T-1}), submit orders to sell to the DP with probability .1125 in the LOB&DP market and .0375 in the DM&DP. If instead they observe a change in the order book only at time $T-1$ and no change at $T-2$ (inv_{T-2} vis_{T-1}), the probabilities are .1500 and .0442 respectively.

Table III - Market Depth, Inside Spread and Tick Size: Time T-2

Panel A - Large Trader - Order Submission Probabilities

$Market$ [b_{T-2}] ($tick\ size$)	$LOB\&DP$ [00]	$LOB\&DP$ [22]	$LOB\&DP$ [11] (.1)	$LOB\&DP$ [11] (.15)	$DM\&DP$
$\varphi(2,\overline{p}_2^B)$	.2912				
$\varphi(2,\overline{p}_1^B)$		.4612	.4168	.3785	.4617
$\varphi(2,\overline{p}^B)$				.1158	
$\varphi(2,p_1^A)$	.2088	.0109	.0832	.1158	
$\varphi(-2,p_{Mid})$		.0279		.0057	.0383
$\varphi(+2,p_{Mid})$		.0279		.0057	.0383
$\varphi(2,p_1^B)$	.2088	.0109	.0832	.1158	
$\varphi(2,\overline{p}^A)$				.1158	
$\varphi(2,\overline{p}_1^A)$		.4612	.4168	.3785	.4617
$\varphi(2,\overline{p}_2^A)$	.2912				

Panel B - Small Trader - Order Submission Probabilities

$Market$ [b_{T-2}] ($tick\ size$)	$LOB\&DP$ [00]	$LOB\&DP$ [22]	$LOB\&DP$ [11] (.1)	$LOB\&DP$ [11] (.15)	$DM\&DP$
$\varphi(1,\overline{p}_2^B)$	.2142				
$\varphi(1,\overline{p}_1^B)$		.4656	.4333	.4054	.4750
$\varphi(1,p_1^A)$	.2858	.0344	.0667	.0946	
$\varphi(0)$					.0500
$\varphi(1,p_1^B)$	.2858	.0344	.0667	.0946	
$\varphi(1,\overline{p}_1^A)$		.4656	.4333	.4054	.4750
$\varphi(1,\overline{p}_2^A)$	.2142				

Table III: Market Depth, Inside Spread and Tick Size - Time T-2. This Table reports large traders' (Panel A) submission probabilities to the LOB&DP at time $T-2$ for the orders listed in column 1. Columns 2 to 5 report results for different states of the book (square brackets) and tick size values (round brackets); column 6 gives the equilibrium order submission probabilities for the DM&DP protocol. Panel B reports the order submission probabilities for small traders. The order submission probabilities reported in this Table allow us to compare different values of market depth, inside spread and tick size.

Table IV - Market Depth and Inside Spread - Large Trader - Time T: vis_{T-2} vis_{T-1} (inv_{T-2} vis_{T-1})

	Order Submission Probabilities: Market Depth				
$Market$ [b_T]	$LOB\&DP$ [22]	$LOB\&DP$ [12]	$LOB\&DP$ [21]	$LOB\&DP$ [11]	$DM\&DP$
$\varphi(2,\overline{p}_1^B)$	.4625 (.4500)	.4625 (.4500)			.4625 (.4558)
$\varphi(2,\overline{p}^B)$			.4250 (.4000)	.4250 (.4000)	
$\varphi(-2,p_{Mid})$	.0375 (.0500)	.0375 (.0500)	.0750 (.1000)	.0750 (.1000)	.0375 (.0442)
$\varphi(+2,p_{Mid})$	.0375 (.0500)	.0750 (.1000)	.0375 (.0500)	.0750 (.1000)	.0375 (.0442)
$\varphi(2,\overline{p}^A)$		.4250 (.4000)		.4250 (.4000)	
$\varphi(2,\overline{p}_1^A)$	.4625 (.4500)		.4625 (.4500)		.4625 (.4558)

	Order Submission Probabilities: Inside Spread				
$Market$ [b_T]	$LOB\&DP$ [00]	$LOB\&DP$ [20]	$LOB\&DP$ [02]	$LOB\&DP$ [22]	$DM\&DP$
$\varphi(2,\overline{p}_2^B)$	.3875 (.3500)	.4000 (.3750)			.4625 (.4558)
$\varphi(2,\overline{p}_1^B)$			.4500 (.4250)	.4625 (.4500)	
$\varphi(-2,p_{Mid})$	.1125 (.1500)	.0750 (.1000)	.0750 (.1000)	.0375 (.0500)	.0375 (.0442)
$\varphi(+2,p_{Mid})$	.1125 (.1500)	.0750 (.1000)	.0750 (.1000)	.0375 (.0500)	.0375 (.0442)
$\varphi(2,\overline{p}_1^A)$		.4500 (.4250)		.4625 (.4500)	
$\varphi(2,\overline{p}_2^A)$	.3875 (.3500)		.4000 (.3750)		.4625 (.4558)

Table IV: Market Depth and Inside Spread -Time T. This Table reports large traders' order submission probabilities to the LOB&DP market at time T for the orders listed in column 1; columns 2 to 5 report results for different states of the book and column 6 gives the equilibrium order submission probabilities for the DM&DP protocol. Values in parenthesis refer to the case where traders only observe a change in the order book at $T-1$, whereas they do not observe any variation at $T-2$ (inv_{T-2} vis_{T-1}). The Table allows for comparisons among different levels of market depth and inside spreads.

Table V - Price Impact and Price Pressure: $b_{T-2} = [22]$

	Large Trader			
$Market\ [b_{T-2}]$	LOB	LOB&PP	LOB&DP	LOB&DP&PP
$\varphi(2,\bar{p}_1^B)$	.4686	.4704	.4612	.4612
$\varphi(1,p_1^A)$		.0296		.0072
$\varphi(2,p_1^A)$	.0314		.0109	
$\varphi(-j,p_{Mid})$			.0279	.0316
$\varphi(j,p_{Mid})$			.0279	.0316
$\varphi(2,p_1^B)$	.0314		.0109	
$\varphi(1,p_1^B)$		.0296		.0072
$\varphi(2,\bar{p}_1^A)$	.4686	.4704	.4612	.4612

Table V: Price Impact and Price Pressure - $b_{T-2} = [22]$. This Table reports large traders' submission probabilities for the orders listed in column 1 for the benchmark (LOB) and for the LOB&DP framework, both with and without price pressure.

Table VI - Market Quality

Book	Estimated Spread			Estimated Depth			Estimated Volume				
	LOB	LOB&DP	% Δ	LOB	LOB&DP	% Δ	LOB	LOB&DP	% Δ	DP	LOB&DP+DP
$b_{T-2} = [00]$											
T-2							.7684	.7967	.0368		
T-1	.2483	.2505	.0089	.7316	.7033	−.0387	1.1450	1.1643	.0169		
T	.2469	.2488	.0077	.7650	.7325	−.0425	1.3074	1.2386	−.0526		
Average	.2476	.2497	.0085	.7483	.7179	−.0406	1.0736	1.0665	−.0066		
Total							3.2208	3.1996	−.0066	.0804	3.2800
$b_{T-2} = [11]$											
T-2							1.2606	1.2669	.0050		
T-1	.1846	.1850	.0022	1.3939	1.3830	−.0078	1.2556	1.2367	−.0151		
T	.2212	.2218	.0027	1.0624	1.0269	−.0334	1.3201	1.2525	−.0512		
Average	.2029	.2034	.0025	1.2282	1.2050	−.0189	1.2788	1.2520	−.0209		
Total							3.8363	3.7561	−.0209	.0925	3.8486
$b_{T-2} = [22]$											
T-2							1.4021	1.3880	−.0101		
T-1	.1469	.1461	−.0054	2.6958	2.6681	−.0103	1.3479	1.3196	−.0210		
T	.1878	.1868	−.0053	1.8326	1.8016	−.0169	1.3474	1.2917	−.0413		
Average	.1674	.1665	−.0054	2.2642	2.2349	−.0130	1.3658	1.3331	−.0239		
Total LOB							4.0974	3.9993	−.0239	.1043	4.1036

Table VI: Market Quality. This Table compares estimated inside spread (columns 2 to 4), estimated depth at the BBO (columns 5 to 7) and estimated volumes (columns 8 to 12) for the LOB and the LOB&DP protocol, and reports the percentage variations ($\% \Delta = \frac{LOB\&DP - LOB}{LOB}$) across different books and periods.

Table VII - Systematic Pattern in Order Flows

	$b_{T-2}=[20]$			$b_{T-2}=[10]$			$\%\,\Delta:\ \frac{(b_{T-2=[10]})-(b_{T-2=[20]})}{b_{T-2=[20]}}$	
Large Trader	LOB	LOB&DP	$\%\,\Delta:\ \frac{LOB\&DP-LOB}{LOB}$	LOB	LOB&DP	$\%\,\Delta:\ \frac{LOB\&DP-LOB}{LOB}$	LOB	LOB&DP
$\varphi(2,\overline{p}_2^B)$	.4132	.3878	−.0615	.3805	.3824	.0050	−.0791	−.0139
$\varphi(2,p_1^A)$	.0692	.0118	−.8295	.1116	.1096	−.0179	.6127	8.2881
$\varphi(-2,p_{Mid})$		.0827						
$\varphi(2,p_1^B)$	.1174	.1139	−.0298	.1679	.1625	−.0322	.4302	.4267
$\varphi(2,\overline{p}_1^A),[\varphi(2,\overline{p}_1^A)]$	[.4001]	[.4038]	[.0092]	.3400	.3455	.0162	−.1502	−.1444
Small Trader								
$\varphi(1,\overline{p}_2^B)$	.4063	.4076	.0032	.3401	.3447	.0135	−.1629	−.1543
$\varphi(1,p_1^A)$	.0776	.0762	−.0180	.1544	.1500	−.0285	.9897	.9685
$\varphi(1,p_1^B)$	.1725	.1570	−.0899	.1567	.1415	−.0970	−.0916	−.0987
$\varphi(1,\overline{p}_1^A)$	.3436	.3592	.0454	.3488	.3638	.0430	.0151	.0128
Average Probability								
$\frac{1}{2}\sum_{j=1,2}\varphi(j,\overline{p}_2^B)$	.4097	.3977	−.0293	.3603	.3636	.0092	−.1206	−.0857
$\frac{1}{2}\sum_{j=1,2}\varphi(j,p_1^A)$	.0734	.0440	−.4005	.1330	.1298	−.0241	.8120	1.9500
$\frac{1}{2}\varphi(-2,p_{Mid})$		.0414						
$\frac{1}{2}\sum_{j=1,2}\varphi(j,p_1^B)$	.1450	.1354	−.0662	.1623	.1520	−.0635	.1193	.1226
$\frac{1}{2}\sum_{j=1,2}\varphi(j,\overline{p}_1^A)$	.3719	.3815	.0258	.3444	.3546	.0296	−.0739	−.0705

Table VII: Systematic Pattern in Order Flows. This Table reports the submission probabilities of the orders listed in column 1 for both large and small traders, as well as the average probability of market, limit and DP orders. Results are reported for the two states of the book $b_{T-2}=[20]$ and $b_{T-2}=[10]$. For each state values are shown for both the LOB and the LOB&DP, and for the difference of the two ($\%\Delta=\frac{LOB\&DP-LOB}{LOB}$). Columns 8 and 9 show how the submission probabilities of each order change moving from $b_{T-2}=[20]$ to $b_{T-2}=[10]$, both for the LOB and the LOB&DP protocol ($\Delta=\frac{(b_{T-2=[10]})-(b_{T-2=[20]})}{b_{T-2=[20]}}$).

Table VIII - Asymmetric Information on the state of the DP - Large Trader

Panel A.1 — $b_{T-2}=[22]$

Type	DP	LOB Uninf	LOB&DP Uninf	LOB&DP&IOI Uninf	Inf [0]	Inf [+6]	Inf [-6]	WA	LOB&DP&IOI$_{vis}$ Inf [0]	Inf [+6]	Inf [-6]	WA
$\varphi(2,\bar{p}_1^B)$		.4686	.4612	.4542	.4667		.4721	.3836	.4581		.4750	.3110
$\varphi(2,p_1^A)$		.0314	.0109	.0021	.0210		.0245	.0086	.0132			.0044
$\varphi(-2,p_{Mid})$			.0279	.0437	.0123	.5034		.1078	.0287	.5105		.1798
$\varphi(0)$									.0287	.0145	.0145	.0096
$\varphi(+2,p_{Mid})$			.0279	.0437	.0123		.5034	.1078	.0287		.5105	.1798
$\varphi(2,p_1^B)$		.0314	.0109	.0021	.0210	.0245		.0086	.0132			.0044
$\varphi(2,\bar{p}_1^A)$		.4686	.4612	.4542	.4667	.4721		.3836	.4581	.4750		.3110

Panel A.2 — $b_{T-1}=[22]\,inv_{T-2}$

Type	DP	LOB Uninf	LOB&DP Uninf	LOB&DP&IOI Inf [+2,(-2)]	Inf [+8 or +4]	Inf [-8 or -4]	WA	LOB&DP&IOI$_{vis}$ Inf [+2,(-2)]	Inf [+6 or +4]	Inf [-6 or -4]	WA
$\varphi(2,\bar{p}_1^B)$		.4750	.4500	(.4750)		.4750	.3427	(.4750)		.4750	.2375
$\varphi(-2,p_{Mid})$			.0500	.5000	.5030		.1517	.5000	.5059		.2528
$\varphi(0)$		.0500		.0250 (.0250)	.0220	.0220	.0112	.0250 (.0250)	.0191	.0191	.0194
$\varphi(+2,p_{Mid})$			.0500	.5000		.5030	.1517	.5000		.5059	.2528
$\varphi(2,\bar{p}_1^A)$		.4750	.4500	(.4750)	.4750		.3427	(.4750)	.4750		.2375

Table VIII: LOB&DP&IOI - Asymmetric Information on the state of the DP. This Table reports in Panel A large traders' submission probabilities for the orders listed in column 1 for four protocols, LOB, LOB&DP, LOB&DP&IOI and LOB&DP&IOI$_{vis}$. For the last two protocols, i.e. the markets with IOI messages, results for informed large traders are shown for the different possible states of the DP, and columns 8 and 12 report for each order type the weighted average (WA) of large traders' submission probabilities. For example, at $T-2$ the weights used to compute the average submission probability of DP sell orders are equal to the unconditional probabilities of the different states of the DP: $\frac{1}{2}.0437 + \frac{1}{2}[\frac{1}{3}.0123 + \frac{1}{3}.5034]$; at $T-1$, instead, the weights are computed by using the conditional probabilities of the different states of the DP. Small traders' order submission probabilities are reported in Panel B.

Table VIII - Asymmetric Information on the state of the DP - Small Trader

Panel B	$b_{T-2} = [22]$			$b_{T-1} = [22]$ inv_{T-2}		
Small Trader	$LOB\&DP$	$LOB\&DP\&IOI$	$LOB\&DP\&IOI_{vis}$	$LOB\&DP$	$LOB\&DP\&IOI$	$LOB\&DP\&IOI_{vis}$
$\varphi(1, \bar{p}_1^B)$	.4656	.4675	.4687	.4750	.4750	.4750
$\varphi(1, p_1^A)$	.0344	.0325	.0313			
$\varphi(0)$				.0500	.0500	.0500
$\varphi(1, p_1^B)$	.0344	.0325	.0313			
$\varphi(1, \bar{p}_1^A)$	.4656	.4675	.4687	.4750	.4750	.4750

	LOB	LOB&DP	LOB&DP&IOI	LOB&DP &IOI$_{vis}$	% Δ: LOB vs LOB&DP	% Δ: LOB&DP vs LOB&DP&IOI	% Δ: LOB&DP&IOI vs LOB&DP&IOI$_{vis}$
Estimated Spread							
T-1	.1469	.1461	.1384	.1311	−.0054	−.0527	−.0527
T	.1878	.1868	.1757	.1633	−.0053	−.0594	−.0706
Average	.1674	.1665	.1571	.1472	−.0054	−.0565	−.0630
Estimated Depth							
T-1	2.6958	2.6681	2.8151	2.9494	−.0103	.0551	.0477
T	1.8326	1.8016	1.9872	2.2026	−.0169	.1030	.1084
Average	2.2642	2.2349	2.4012	2.5760	−.0130	.0744	.0728
Estimated Volume							
T-2	1.4021	1.3880	1.2346	1.0907	−.0101	−.1105	−.1166
T-1	1.3479	1.3196	1.1849	1.0538	−.0210	−.1021	−.1106
T	1.3474	1.2917	1.1695	1.0465	−.0413	−.0946	−.1052
Average	1.3658	1.3331	1.1963	1.0637	−.0239	−.1026	−.1109
Total LOB	4.0974	3.9993	3.5890	3.1910	−.0239	−.1026	−.1109
DP		.1043	.5837	1.0330			
Total	4.0974	4.1036	4.1727	4.2240			

Table IX - Asymmetric Information and Market Quality - $b_{T-2} = [22]$

Table IX: Asymmetric Information and Market Quality. This Table compares estimated inside spread, depth at the BBO and volumes for the four following protocols: LOB, LOB&DP, LOB&DP&IOI and LOB&DP&IOI$_{vis}$ Results are reported for $b_{T-2} = [22]$ and across different periods. Column 6 shows the comparison for the three indicators of market quality between the LOB and the LOB&DP markets (% $\Delta = \frac{LOB\&DP - LOB}{LOB}$), column 7 reports results for the comparison between LOB&DP and LOB&DP&IOI (%$\Delta = \frac{LOB\&DP\&IOI - LOB\&DP}{LOB\&DP}$), whereas column 8 presents the comparison between LOB&DP&IOI and LOB&DP&IOI$_{vis}$ (%$\Delta = \frac{LOB\&DP\&IOI_{vis} - LOB\&DP\&IOI}{LOB\&DP\&IOI}$).